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Teaching Third Grade Math : A Learning Experience

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This study was based on my experience as a new teacher, teaching math to a group of third graders. The purpose was to examine my methods of planning and teaching in order to better understand how I was learning and changing my views on teaching math. I gathered information by reading educational theorists, observing the children in my group, recording anecdotes and writing about my planning process. I gained inspiration from readings of Piaget's and other cognitive theorists, I gained insights from the ways my children responded to my lessons and I discovered that using manipulatives, examples that the children could relate to and a touch of fun were the foundations of my teaching philosophy.

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Chapter I - Introduction

I went to grammar school in the 1970's, and back then I didn't understand a lot of the things I "learned", especially when it came to math. I was always better at addition than subtraction, but no one showed me how addition and subtraction were related. I don't remember learning about number families, and multiplication was never five groups of two, it was 5 times 2 ("times" being the official name for "x".)

I was able to get through math with out much trouble because I managed to make up my own rules and reasons for things. Being a fairly good rote memorizer, whatever I didn't understand I could at least produce on cue, and by following the memorized steps I'd get through the problem.

I bombed out in algebra because I never understood the order of operations and I made a lot of "careless errors" in my calculations. Basically I had no idea what was going on.

In high school geometry I was safe again, lots of memorizing. With the onset of trigonometry, though, I lost all confidence in my mathematical ability and came down with a serious case of math anxiety.

Three years after my last dealings with math I started volunteer work in a kindergarten classroom. I learned about one to one correspondence and collages of five things, and math that connected to real life and other topics in the curriculum. One of my favorite math activities was "snack math". We had an attendance chart and as soon as everyone was seated at the snack tables the teacher would ask how many people were there that day. Then we'd count the apples or peanuts we had for snack together, and the children would count or "guestimate" how many or how many pieces we'd have to divide the snack into so that everyone would get the same amount. The amazing thing was hearing their explanations. They all had reasons, many of them different, but they were all hearing each other and talking about math in an everyday context.

I worked in that school for three and a half years in the kindergarten and 1st grades and was introduced to manipulative materials, which began my re-education in mathematics. Once I learned some of the ways to use Unifix cubes, pattern blocks, Dienes blocks and Cuisenaire rods I became a devoted fan and staunch supporter of hands-on materials. (see appendix)

I left that school for the Bank Street College of Education and an internship at another school and took my first math class for teachers. Meeting a class full of teachers who had math anxiety like I did made me feel a)worried, b)hopeful and c)determined to re-learn math so that I could teach children how to understand.

This year I am assisting in a third grade class where I am responsible for teaching a math group consisting of three boys and six girls between eight and nine years old. Last year I had experience as an intern for half of the year in another third grade classroom where I followed my head teacher very closely in terms of planning math activities and didn't feel the responsibility as keenly as I do this year. This year I am more actively involved and concerned with the math program and even though I follow set topics in mathematics I have the freedom to be creative and to have the responsibilities, at least for math time, of a head teacher.

Chapter II - Research Methods

Between October 29th and December 11th I concentrated on my planning techniques, the lessons themselves, observing and recording the reactions of the children and making note of insights that occurred to me during this time. In creating lessons I relied on my previous experiences as a student and teacher, information and advice from my head teacher, ideas set forth by math education theorists, and my imagination.

During this study my primary sources were the children in my math group. In order to keep track of what happened during our lessons, I amassed a collection of jottings on slips of paper. I transcribed these into a journal every day during my break or after school. I split the journal into two sections, "Notes" and "Ideas" and kept my observations of the children in the "Notes" section. My preparations for lessons (e.g. thoughts, rationales, fears), went into the "Ideas" section. I found that if I talked to someone about the lesson or what interesting things happened, the experience stayed with me longer. Going back to the table where we worked or picturing the scene in my mind also helped me remember details.

Chapter III - Literature

Through reading that I did for class at Bank Street and additional reading I have done on my own, my perceptions about learning math and teaching it have become clearer. Key mathematics theorists that I have studied are Jean Piaget, Constance Kamii, Neil Davidson, Marilyn Burns, Jerome Bruner and Richard Skemp. Some of the processes and teaching methods, Piaget's for example, have shown up right away in my lessons and interactions with my math group. Other ideas, such as Constance Kamii's discussion of autonomy in the classroom, I find I'm still digesting.

I found a liking for Piaget in Ed Labinowicz's books, The Piaget Primer and Learning From Children. The clear depictions of children performing Piagetian tasks were models for me of how to work with manipulatives and how to talk to children about what they're thinking and learning. Piaget places the 8 and 9-year-olds I teach in the Concrete Operational stage of development where they are able to engage in logical thinking but are limited to physical reality (Labinowicz 1985, Labinowicz 1980). Children perceive the world differently than adults do--from their developmental point of view. Labinowicz summarizes that, "we see what we understand rather than understand what we see." (Labinowicz 1985).

Piaget's Constructivist viewpoint stresses that a child learns by interacting with his environment and the objects in it as well as building concepts on previous experiences. As much as possible I include manipulatives in my math lessons. From the children's work with Dienes blocks, for instance, I move them on to pictorial representations, and then finally to symbols so that they have a base to build on and an understanding of where the concepts have come from. I try to understand how my children think and what they know about math by asking them and by looking at their errors. I saw a lot of "subtracting up" when we were learning about regrouping this year. I was able to help my students by having them talk through their methods to become more aware of their problem solving strategies or by pointing out patterns in their answers if they couldn't remember how they got them. This is when a mini-lesson with others who were having similar problems, or a one-on-one using manipulatives would be helpful, depending on the child and the topic. A phrase in Learning From Children that I feel is especially meaningful is, "A child never makes mistakes. He gave the right answer to a different question." (Labinowicz 1985)

I read parts of Constance Kamii's book, Young Children Reinvent Arithmetic--Implications of Piaget's Theory, and found her discussion of different forms of autonomy thought provoking and inspiring.

In Chapter 3, Ms. Kamii describes two kinds of autonomy. There is moral autonomy which is governing one's own behavior or sense of right and wrong by respecting or acknowledging others' viewpoints. Intellectual autonomy is governing one's ideas about true and false by taking into account other explanations. This means self-correcting as well as sticking to one's own answer if there is personal conviction of its truth.

These two forms of autonomy are things I strive to achieve in the classroom and an important part of the message that teachers should send to their students: that everyone should listen to each other and that everyone should be heard.

Ms. Kamii talks about autonomy as the aim of education, which she says is fostering a classroom environment in which self-control and educational risk-taking can co-exist. This would be taking the dictatorial, punitive power from the teacher and allowing children to create their own ideas and learn from their mistakes. These are goals that I hope always to strive for. (Kamii 1985).

For background on cooperative learning I turned to Neil Davidson and Marilyn Burns. Neil Davidson explains the reasoning behind cooperative learning by pointing out that children have lots of energy which is often stifled when they are made to sit and listen to a teacher or participate one at a time. Cooperative learning allows a whole class to work actively at the same time. Mr. Davidson also points out the need children have for communication and interaction with their peers. Cooperative learning allows children to work together, question and help each other and improve peer relations. Cooperative learning has many positive aspects. Working in a group instead of in competition with others enables all the members to be involved and learn the concepts. Since there are objective answers with different solutions, strategies and reasoning can be discussed and children can be exposed to different ways of approaching math problems. Cooperative learning fosters creative thinking and enables a group to solve problems together that the individuals might not be able to solve alone. (Davidson 1990).

Marilyn Burns developed The Math Solution--a teacher training program to teach math so that it makes sense. This involves teaching students how to think mathematically so that they can analyze and solve problems. Ms. Burns believes, as do Piaget and Bruner, that children must actively put together mathematical ideas through experience. She says that physical experience with materials is one important factor for learning. Developmental standing is another--whether the children grasp the concepts at that moment in time. Social interaction is also important to Marilyn Burns, as to Neil Davidson. Interaction allows children to be exposed to different strategies, points of view and methods of working (e.g. in groups, not just individually).

The Math Solution discourages teachers from dividing math into separate topics and encourages teachers to teach math concepts as interrelated threads that weave together to form mathematical understanding, a kind of math sense.

The main thrust of The Math Solution is to teach teachers how to use cooperative, small-group learning in their classrooms. The three basic rules in Ms. Burns's suggested groups of four are: "1) You are responsible for your own work and behavior, 2) You must be willing to help any group member who asks and 3) You may ask the teacher for help only when everyone in your group has the same question." (Burns 1984 from Davidson 1990). These rules take the focus off of the teacher and enable the children to work with each other interdependently.

In a book called Children Learning Mathematics--A Cognitive Approach to Teaching, Emma Holmes combined several different cognitive theories to create the Cognitive Model for Guiding Learning of Elementary School Mathematics. This theory promotes cognitive processing by suggesting that teachers provide thought-producing questions which ask their students to compare, classify, infer and relate objects and ideas. Another part of Ms. Holmes's theory is bound in intrinsic motivation--motivation from within (like Ms. Kamii's and Piaget's autonomy). Ms. Holmes stresses the importance of intrinsic motivation for understanding, learning and success. She also stresses the importance of being sensitive to individual differences that students have in ability and learning style.

Two cognitive theorists Emma Holmes drew from were Jerome S. Bruner and Richard R. Skemp. Jerome Bruner believed that language and learning were closely bound, which figures into my work quite a bit because we do a lot of discussing, explaining, listening and questioning. Richard Skemp believed strongly in the use of physical examples, then pictorial representations before pencil and paper work (Holmes 1985).

Chapter IV - Findings

In addition to literature, I obtained help in lesson planning from my head teacher when I was planning lessons for virtually the first time, this Fall. Every day after school, in the month plus that I did this study, my head teacher, Irene and I would sit down and talk about my math group and the work for the next day. In that time we covered place value, renaming in addition and subtraction, checking addition with subtraction and vice versa, and money. Irene had a system for math instruction: manipulatives to start a topic, board work and paper and pencil work following. I talked to her about using manipulatives because I was really excited about working with them and she assured me that I'd have lots of opportunities to use them. Irene never insisted that I teach a lesson her way, but she was always willing to show me a method that had worked for her in the past or to do a little demonstration for me if I couldn't wrap my brain around a concept that I had learned differently.

Three components I always tried to include in my lessons were manipulatives, examples the children could relate to, and something fun. Irene usually told me the concepts I should include and showed me sheets they could work on after the lesson but the rest was up to me.

On October 29th, my math group, "XY" was learning about the concept of thousands in the context of place value using Dienes blocks. Previously we had worked with the ones cubes, the tens sticks (longs) and the hundreds flats. All of the children had been introduced to these materials in second grade and a few of them remembered the thousands cube.

I gave out some flats randomly and put the bin with more in the middle of the table. I gave them thousands cubes and asked them to make a shape the same as "the big cube" using the flats. When everyone had finished I asked them how many flats they used. They were in agreement that they used ten.

Next I asked them what the value of their construction was, how much they had. To figure it out I asked them how much one flat was worth. Josh counted each cube by ones on the face of the flat and came up with a number that was under 100 and not rounded. He seemed surprised by his own answer and said he wasn't sure if that number was right. I suggested that he could cover the flat with tens sticks and count by tens to check his first answer. He did so and found that the flat's value was 100.

I asked again about the value of the cube. Some of the children already knew it was 1,000, but Josh wasn't convinced. He studied the cube and said he thought it was worth 500. (I had read Labinowicz's example of this lesson and was ready for this kind of answer). I asked him what his method was for figuring out 500. He picked up five flats and held them onto the sides of the cube. I asked him if there was one flat on the bottom and he turned the cube upside down, said yes and changed his answer to 600. I asked him to hold the six flats in one hand and the cube in the other and asked him which was heavier. He chose the cube, and when I asked him why it might be heavier he supposed that there were cubes inside.

We went back to those ten original flats that were the same shape and size as the cube and we counted out loud together by hundreds until we had unpiled the flats and reached 10 hundreds, which a number of the children knew was also 1,000.

Our next question to tackle was how many tens sticks in the cube. How could we figure this out without having to use individual tens sticks? Sarah pointed out that there were ten sticks in a flat and that we could count by tens so we did. For every ten tens I'd take a flat off of the model they'd made until we reached 100.

Knowing what we now knew I asked them how many ones cubes in the 1,000 cube? Josh immediately said 1,000 because, he explained, there were 1,000 little cubes in the big cube, ones are what you count by. He told me that ten flats were 1,000, 100 longs equalled 1,000, and 1,000 ones equalled 1,000, they all came out to 1,000. Why I asked? Because they are all the same as the big cube, they told me. Do you mean equal I asked? Yes, they responded.

I passed around more of all of the kinds of Dienes blocks, asked them to choose partners and told them they could combine their blocks using all or some to show a number in the thousands. Josh and Jimmy started piling their like blocks making one big connecting shape. I asked them to set it up from left to right so that it would be easier for us to read the number. I realized that I was dangerously close to having to rename things when we wrote up how many of each block was used and wrote out the values of their block collections (we hadn't gotten to renaming yet).

As it turned out the boys decided not to use all their blocks because, they said, "It got too complicated." I think they had a lot of blocks and they were having trouble keeping track of them. The two girls, Sarah and Katharine knew exactly how many of each they had and the values. I set these up horizontally on the blackboard: 2,000 + 1400 + 30 + 6 as they told them to me and then I put them vertically and the girls added them up.

Josh's experience with the thousands cube was interesting because I could see how he was thinking, and by using the concrete materials he was able to check himself and change his ideas to fit the reality he was seeing.

On November 4th I was thinking of ways for the children to discuss their strategies with each other, show me they understood double digit place value and have a little fun. I decided to write several different kinds of problems on the board, have them each go up to the board and answer one, and explain how they got their answer. I knew they all loved to write on the board and that their strategies would be different and, I hoped, helpful to each other. It would also give me an opportunity to observe their thinking.

The next day, Jimmy went up to the board to answer 8 + 6 and decide which digit of the answer belonged in the tens column and in the ones column. I asked, "How did you figure this out? What gave you the clue about how to solve it?" He told me that he knew 8 + 6 was 14 because he took 2 from the 6 and added it to the 8 making 10 and then added the 10 to the 4 left.

Abbie's problem was to take 17 and put the digits in the space for tens and the space for ones. She wrote 10 tens and 7 ones and after a prod from her friend, Mary, changed the 10 to a 1. I was more interested in the incorrect answer so we tested 17. I had Abbie take 17 ones cubes out of the bin and asked her if we had more than 10, enough to trade. She replied that we could trade 10 and then proceeded to miscount ten, picking up 9. I asked her if she'd count them one more time and she did, realizing her error, and picking up one more. We traded and I gave her a ten stick. I asked her how many ones she had left. She counted 2 and stopped and said 7. When I asked her how she knew 7, she told me that she had 17, she traded 10 and she knew that she'd have 7 left. I asked her how much she'd have if she had 10 tens sticks. She said, "100 (indicating the answer she had originally written on the board) so that was wrong." "But what made you think 10 the first time," I asked. She didn't know. I asked her if maybe she had been thinking about trading 10 ones cubes and that's where the 10 came from. She nodded her head but I realized that next time I should encourage her to figure out why instead of jumping in with an answer for her.

On November 7th I decided that along with the problems that I'd put on the board for the group to do together, I'd include some that I would do incorrectly the way that I'd seen some of them do. I asked them not only to correct them but to tell me how I had figured out those answers. Of course it's easier to notice someone else's mistakes, and making them my mistakes instead of a classmate's gave them the freedom to pick the mistakes apart and figure out how they were made. The children were very enthusiastic, calling out the correct answers and volunteering to come up to the blackboard to point out what was wrong. When I asked them how I had figured them out this way they were able to articulate the process and even though some of the prime offenders weren't the ones who caught their own styles of mistakes, other children explained them so that everyone at least heard it.

On November 14, I brought in a piece of real-life math that came off a bag of Smartfood cheese popcorn. I had been munching away the day before when I noticed that on the upper right hand corner of the bag they boasted, "Over twelve billion, seven hundred ninety-four million, twenty-three thousand, six hundred twelve kernels popped." I mounted the front of the bag on construction paper and wrote, "Try the Smartfood Challenge. Can you write this amount in numbers?" We had been working on place value but only through the thousands, so I walked through the places with three of my math students who were interested. I told them there were several patterns to look out for. I showed them 1's, 10's, 100's on the board. Then I showed them 1,000's, 10,000's, and 100,000's. I asked them what patterns they saw. They noticed more zeros first and I pointed the 1, 10, 100 pattern out to them. I told them the names million and billion and they dictated the rest of the numbers to me.

Then Sarah wrote the Smartfood number starting with the ones, because she said that the lower numbers were more familiar to her. We talked about the 23 thousand which didn't have a number in the 100 thousand place and Sarah was the first one to solve the challenge.

Jimmy tried the challenge and used 4 pieces of paper until he was satisfied. He was still missing the zero in the 100 thousands. I told him about the commas every three digits. He wrote 1,279,423,612. I told him that the 794 million went together in a group. What could he do for the 23 if there needed to be another digit? He suggested zero but immediately asked where you put it. "Try it out," I told him. He put it after the 3 and read it, " 230 thousand, no. In between? 203 thousand, no. Before? 023 thousand, yes!" The biggest problem was that silent zero.

I'd like to do more real-life math, finding math in the numbers of everyday life, numbers in your address, phone, packaging (like six-packs). This might have worked nicely in small-group work. I try to be on the look-out for real-life math so that I can integrate it into the curriculum.

Three weeks after our unit on place value, on November 19th, I was working with half of the group on subtraction with regrouping using Dienes Blocks. I had given out the mats with space for ones cubes, tens sticks and hundreds flats and asked them to show me 40 using tens sticks. I asked them to trade a tens stick for ones or show more ones. I asked how many they had now and got the answer, 40. Jimmy was disturbed because we had 10 cubes in the ones column and when we did addition we had always traded 10 or more. I explained we were doing the reverse of trading ones for a tens stick and we wanted to "show more ones," so in this case it was okay to have ten ones. I told him he had a good point and I could tell he was really thinking about what we were doing.

We practiced showing more ones and taking away single digit numbers and on our third example I had them set up 54 so that we could take away 6. In setting up, Mary decided to show 3 tens sticks and 24 ones. She told me it would be easier to subtract her way. I let her do it because I knew she understood the other way and had anticipated trading tens sticks for ones. I split my lesson in two and talked to Mary about how many she had and to the other three about showing more ones and how many they had afterward. Mary subtracted her 6 and then looked at the blocks in front of her and suddenly was unsure she had started with 54. I told her to put her 6 back and count her ones again. She had 24 and added that to her 3 tens sticks and was satisfied that she had 54. So much for being sure she understood. I think it was difficult for her to do her thinking with four other people discussing another method. I think it was good for her to try something and have my support, and that's why I encouraged her to recount her blocks so that she wouldn't feel that she had failed. She has a good idea: it just wasn't the best time to try it out.

Mary joined us in showing more ones, and my focus turned from her to Josh, who was starting to have troubles. I noticed a pattern. When we were subtracting from round numbers he was fine, but if we had a number like 23, with 3 cubes in the ones column to start with, everyone would show more ones and have 13 ones and Josh had 10. When I asked him how that occurred he stated that he had traded a tens stick for 10 ones. I reminded him that he started with 3 ones on the mat and that the 10 ones were separate from those 3. I suggested he separate them when he counted but he didn't want to, and it happened again. This time he could tell me what happened and fix the problem.

We did some more examples and I asked if anyone saw a pattern in the tens and ones when we did these trades, or "showing more ones." I was looking for something along the lines of, the tens go down, and the ones go up. Sarah gave me a much better and more precise answer. She told me that the ones went up by 10 and they became a two digit number and the tens went down by one each time.

I like working with the manipulatives because you can always reverse the process with the blocks and figure out what made you go wrong. With pencil and paper, if you don't show all your computation work, it's often hard to figure out what you were thinking at the time you did it.

For fun in the beginning of December I made up a game that I called "Double Rename-It". We had done renaming by showing more ones and showing more tens and the next step was to do both in one problem. To make it more interesting, I gave each child two numbers. A single digit, e.g. 3, and a double digit counterpart in the teens, e.g. 13. We would do some examples on the board together, and whenever we crossed a number out to borrow and rename, the owner of the new number had to call it out. This went for answers that appeared in each column as well. If they didn't call it out right away, I made a buzzer sound and the penalty was to give four math facts about one of their numbers. They really liked it and everyone was involved. I had a great time too!

Using examples that the children can relate to has been an important part of my lessons. Josh told me one day that 0 - 5 = 5. I said to him, "Say you have no baseball cards, zero. Can you take 5 of them away?" He said no but he knew there was some trick with zeros. I said, "O.K., now you have 5 baseball cards and you take zero away. Can you do that? What will you have left?"

"Oh I get it," Josh burst out, "you can take zero away from something but you can't take a number away from zero." It made sense to him.

Whenever we've done work with word problems I've made them up myself, using the children's names and interests, I think they listen and remember them better when the problem has something to do with them.

I also believe in using physical examples and pictures first before using numerical symbols. On December 4th I was planning a lesson on renaming with zeros for the next day. Before launching into a potentially confusing series of crossing out, borrowing and renaming, with numbers like 400, 100 and 600, I thought--we refer to renaming as borrowing, so why not tell a story about borrowing with people, objects and pictures to convey the idea of borrowing with numbers.

That's how I came up with the story of borrowing milk: "I was getting ready to have a bowl of cereal for breakfast when I realized I had no milk. I went next door to Lauren's house to borrow some milk, but she had no milk. Jimmy lived next door to Lauren but I didn't want to borrow milk from him myself because I've heard that he's cranky in the

morning. So Lauren went to Jimmy's and he had plenty of milk because his mom had just gone to the supermarket the day before. He gave Lauren some milk and then she gave some to me."

I made a picture on a large piece of paper of 3 houses labeled Jimmy's House, Lauren's House and My House. Then I drew glasses of milk (full and empty), which I taped on so that they were removable, and a picture of a bowl of cereal and spoon. I hoped that if they understood the story and could recall it, then they could apply it when we used numbers. When we did some board work I referred to the numbers as "me without any milk for my cereal" and "Jimmy with 5 glasses of milk". They certainly enjoyed the example and I think it helped some of them. There were errors, of course, but after all my story was simpler than working with the real numbers.

Chapter V - Conclusion

Since the end of this study I've brought in groceries for work with money, T.V. schedules for work on time, and more word problems about winning Superbowl tickets and hours logged in Nintendo playing. This is real life, these are things the children relate to and this is fun math!

Doing this study and reading my journal after some time has passed has given me insights into my teaching and lesson creating styles that I might not have gained otherwise. I didn't realize, for instance, that I sum up the lesson's focal point myself instead of asking the children what they learned or got out of it. I think I know why I do that: I don't trust that I've given out the information or clues clearly enough so I give a brief summary to make sure everything has been said. In the future I'm going to make a concerted effort to ask the children for their version of the concept we've just worked on. I'd like to let them teach and learn from each other.

I have also found that I sometimes ask questions one on top of another without pausing to let the children think in silence. While I know that some children need more time, I also worry about the children who don't, who get bored. Somehow I feel that as long as I'm talking they'll stay with me. I don't feel comfortable with the silences yet. When I'm reading a story aloud to a group, I know with assurance that if I stop, they'll be on the edge of their seats waiting to hear the next word. I don't have that confidence yet in teaching math. But every day that I teach a lesson, I'm gaining experience, I'm picking up hints about what works and what really doesn't and no one has died yet of a bad math lesson.

I know very well that I need to plan ahead. I'm not a person who can "wing it" comfortably, so I take notes during my meeting with Irene, I write up my lesson on a legal pad and then I transfer it to a 5" x 8" index card which I bring to class with me. I held the index card in my hand in the beginning of the year, and now I glance at it at the beginning of the lesson and put it down for the rest of the time--unless I have a number example written down.

I've learned that excitement is contagious, and when I feel like I've got a terrific lesson and I present it with enthusiasm, the children are enthusiastic too. I've learned that I care a great deal about children understanding and feeling competent in math. I've learned that there are many levels of comprehension, and that even if a child doesn't "get it" the first time around that doesn't mean I"m a bad teacher.

On the first day of school while we were lining up for something, out of the blue, Josh burst out with, "I hate math!" When I asked him why, he answered, "First of all I'm not very good at it and second, it's boring." I figured he had been carrying that bit of news around al day and could use some reassurance. I told him that I had had lots of trouble with math myself and that it got the most boring for me when it didn't make sense and I couldn't understand what to do. I also told him that I was going to work really hard this year on making math clear and understandable so that it wouldn't be boring. So far, I think I have.

Appendix

Unifix cubes - Plastic interlocking cubes that come in assorted colors which can be used for counting as well as many other mathematical operations.

Pattern blocks - Wooden blocks in various colors and shapes e.g. yellow hexagon, green triangle, red trapezoid, blue rhombus. These can be used for work with fractions, geometry and making patterns among other uses.

Dienes blocks - Also called Base Ten Blocks or Powers of Ten Blocks these come in 1 cm cubes (units), 10 cm rods (longs), 10 cm x 10 cm squares (flats) and 10 cm x 10 cm x 10 cm thousands cubes. These are commonly used for place value work and regrouping in addition, subtraction, multiplication and division.

Cuisenaire rods - Wooden rods in assorted colors from 1 cm to 10 cm long used for building, number relationships, and fractions to name just a few.

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