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Learning mathematics becomes more effective when teachers leverage their students' mathematical and everyday knowledge as resources for instruction. Thus, tasks that reveal these forms of knowledge would be especially useful to teachers. Unfortunately, such tasks are hard to find and even harder to create. Consequently, we developed a collection of mathematical tasks that we hoped would elicit “children's multiple mathematical knowledge bases (i.e., the understandings and experiences that have the potential to shape and support children's mathematics learning—including children's mathematical thinking, and children's cultural, home, and community-based knowledge)” (Turner et al., 2012, p. 68). These tasks proved to be productive and took on two forms, one of which called on students to provide mathematical and real-world justifications for decisions, and one of which used a community issue as a “contextual scaffold” for linking students’ community- and school-based knowledge.
Cultivating a Space for Critical Mathematical Inquiry through Knowledge-Eliciting Mathematical Activity

Debasmita Basu & Steven Greenstein

As teachers, we know that learning is more effective when instruction connects the mathematics we aim to teach and the home, community, and cultural knowledge students bring with them to school. Indeed, classrooms can only operate as venues for critical mathematical inquiry if instruction draws out and builds on this knowledge. We also realize that the benefits extend beyond making learning more effective. Engaging this knowledge also helps us cultivate the kinds of caring relationships that nurture students’ sense of belonging (Horn, 2017) and contribute to the myriad ways we experience the joys of teaching and learning.

It is one thing to know the benefits of leveraging what Turner and her colleagues refer to as children’s multiple mathematical knowledge bases (MMKB) (Turner et al., 2012)—or “the understandings and experiences that have the potential to shape and support children’s mathematics learning—including children’s mathematical thinking, and children’s cultural, home, and community-based knowledge” (p. 68). It is quite another to undertake the considerable effort required to elicit this knowledge from students. While teachers tend to believe the effort is worthwhile, they often find they lack the time to do it (Gonzalez et al., 1993, p. 1390). Thus, tasks that reveal students’ multiple mathematical knowledge bases can be useful to teachers who wish to leverage their students’ knowledge as resources for more effective instruction. However, such tasks are hard to find and even harder to create.

In this article, we share the findings of a project we undertook, which we titled knowledge-eliciting mathematical activity, or KEMA. Our goal for the project was to develop task design principles that teachers could use to reveal their students’ multiple mathematical knowledge bases. We present some of the tasks we found to be effective along with some of the things we learned, aiming to offer guidance to teachers to develop their own tasks. We believe the principles we used to design these tasks will be useful to teachers who wish to enact a responsive mathematics pedagogy that is deeply connected to their students’ bases of mathematical knowledge.

Participants

This study was implemented in a community charter school in a low-income, urban setting in Newark, New Jersey. The school has an enrollment of 110 students, and 92% of them are eligible for free or reduced-priced meals. Many of the students are either immigrants or first-generation children born of immigrants from countries including Brazil, Cameroon, Ecuador, Ghana, and Nigeria. One mixed-grade class of 15 elementary and middle school students ranging in age from 9 to 13 years old participated in the study. The classroom teacher is a mathematics teacher; he allowed us to assume control of the classroom while we were there. We asked for this permission so that we could do our best to assume the role of teachers who had much to learn about their students.
Phase 1: Knowledge-Eliciting Mathematical Tasks

Every student brings a range of everyday and out-of-school knowledge with them to school, and these culturally determined ways of knowing frame their perspectives and determine what they see (Schoenfeld, 1992). Our first attempt to develop mathematical tasks that we hypothesized would elicit children's multiple mathematical knowledge bases involved the use of Would You Rather? tasks, which we found at the website, www.wouldyourathermath.com (Stevens, n.d.).

Would You Rather? tasks offer two options to students and call on them to choose one and justify their decision. By their nature, the tasks invite students to use mathematics to craft their justification. We were drawn to these tasks because we have found them to be richly revealing of students’ mathematical knowledge. A sample appears in Figure 1.

We saw the promise in revising and modifying these tasks so that they had both mathematical features and real-world contexts that we thought students would relate to. In this way, they could reveal students’ cultural, home, and community-based knowledge, as well as their mathematical knowledge.

Figure 2 presents two of the tasks we modified. The one on the left poses the question, “Would you rather live a 10-minute bike ride from school or a 5-minute bus ride?” This task could be answered by relying solely on the mathematics—that is, 5 is less than 10—but we also imagined that it would engage other forms of knowledge. Some students may prefer to ride a bike, some may enjoy the company of friends on the bus, and some may be all too familiar with the effects of traffic and how it varies, depending on the time of day.
When we implemented the task, students emphasized the waiting time for a bus and the lengthy pick-up and drop-off times, and expressed concerns about local traffic conditions. One student argued, “I will choose the 10 minutes by bike over a bus, because sometimes there could be lots of traffic and in a bus, I have to wait for a longer time…. Riding a bike is also like exercise.” Another student preferred the bus, because “it is quicker and gives you more opportunities to chat with your friends.”

The task on the right in Figure 2 is also revealing. Students were shown pictures of two cakes, the first a one-layered cake and the second, a five-layered cake, and asked, “Which one would you rather share with your family?” Students picked one of the two cakes and then provided justifications for their decisions. A student who chose the five-layered cake explained, “If you have big family, you can eat it all. It will be like no leftover.” Another student chose the one-layered cake for a reason that also related to sharing. She added, “Say, like, if you have like a small family, it would be like – I realize that the pieces are not same, but you can share with your family.” These students’ choices reflected knowledge of sharing that is rooted in both family knowledge and mathematical knowledge (as partitioning).

Phase 1 Results. Through our design and implementation of modified Would You Rather? tasks, we sought to elicit students’ multiple mathematical knowledge bases. Some of our tasks failed to reveal much knowledge at all. For example, one task offered the option, “Would you rather walk to the grocery store or walk to the library?” One student expressed a desire to go to the library in order to “feed her brain knowledge,” while another realized that although he was given a choice, he would need to go the grocery store at least once a week. Still, another preferred to go to the grocery store rather than the library because at the grocery store, “you can talk while you get things you like.” Similarly, another less productive task offered the choice between collecting loquats or cleaning trash from the beach. Students didn’t know what a loquat was, but they assumed it was healthy. And the beach reference didn’t fully resonate, although some students did express the importance of keeping it clean. That task told us something about what students didn’t know.

On the other hand, other tasks, including the ones we presented above, elicited students’ mathematical knowledge related to fractions, rate, ratio, and area. This is to say that we used those tasks in conjunction with teacher discourse moves like waiting, probing, and revoicing to press students for the forms of knowledge we sought to reveal. For instance, they couldn’t just say, “I’ll take the cake on the right because it’s prettier.” These tasks also provided a window into students’ experiences traveling to school, their disposition toward sharing among friends and family members, their food preferences, and the concerns they have about the costs of things. For instance, in the cake task, one student chose the one-layered cake and added, “I chose the first one, because, to be honest, like around here, like a lot of people don’t have a lot of money and they wanna save on other things. So I chose the one that looks like it has the lower price.”

To the extent that we sought to design tasks to elicit mathematical knowledge, we were delighted. That said, we realized that much of what we learned about these students is what we might expect to learn from any group of students. That is, we didn’t know, for example, whether it was reasonable to attribute
the disposition to shared cultural norms. In fact, our findings made us question just what constitutes
cultural knowledge. We had thought of it as some sort of static trait attributable to a particular group of
people. However, it may be better understood as intersectional and related to a variety of ways people
experience the world, their home, their school, their faith, and their social networks. This is a question
we're still pursuing. We wondered whether there might be something missing in the design of our tasks
that was preventing them from evoking the diverse forms of knowledge we sought to assess.

We were also disappointed that the tasks failed to generate much in the way of productive whole-class
discussion (Stein, Engle, Smith, & Hughes, 2008). We initiated the prompts, students responded, and we
struggled to find follow-up opportunities that would generate student-to-student conversation. There
seemed to be no reason for the students to listen to, talk to, or respond to each other. As a result, there
were missed opportunities for students to elaborate on the knowledge they shared and for us to make
connections across their responses. That precluded us from identifying patterns in their experiences
which we could use to make claims about their collective knowledge.

We sensed that we had made good progress toward developing strategies for eliciting students' multiple
mathematical knowledge bases but that there was more potential to realize. Consequently, we decided to
take a new approach to task design in Phase 2 of the project.

Phase 2: Community Issues as Scaffolds for Mathematical Learning

Students in school often fail to find the relevance of what they’re learning. In particular, the kinds of prob-
lems they solve in school often have little to do with the kinds of problems they need to solve in every-
day settings outside of school (Lave, Smith, & Butler, 1988; Roth & McGinn, 1997). Science education
researchers Bouillion and Gomez (2001) framed this disconnect between out-of-school knowing and in-
school learning in the form of the following challenge:

A challenge facing many educational institutions, especially those in urban settings aiming to serve
culturally and linguistically diverse populations, is the disconnect between schools and students' home
communities. Schools are in communities but often not of communities. (p. 878)

In order to remedy the disconnect, Bouillion and Gomez developed an instructional approach known as
“connected science” that bridges the real-world problems students face in their communities with the
science content they are expected to learn in school. Connected science uses real-world problems as
“contextual scaffolds” for linking students’ community-based knowledge and school-based knowledge—
or what we’ve been referring to as students’ multiple mathematical knowledge bases.

The Research Context

We implemented Phase 2 at the same school in Newark, New Jersey, in March 2018. At the time, Newark
was one of the twenty finalist cities being considered by Amazon.com for the location of its second
headquarters, or HQ2. It wasn't until November 2018 that Amazon announced two new locations for
HQ2: Long Island City, in the Queens borough of New York City, and Arlington, Virginia. In the lead-up to this announcement, the twenty cities were involved in intense analyses of the potential costs and benefits of having Amazon's second headquarters.

Amazon promised that they would spend around $5 billion in construction costs on HQ2 and bring in 50,000 new, high-paying jobs. The situation in Seattle, Washington, where Amazon's first headquarters is located, had some citizens of Newark feeling optimistic about the potential benefits if Amazon were to move to the city. From 2010 through 2016, Amazon contributed $38 billion to Seattle's economy, and each of those dollars generated an additional $1.40 in the city's economy. Some Newark citizens were not only imagining thousands of new jobs, they also foresaw thousands of additional jobs and tens of billions of dollars in additional investment in the communities surrounding Newark. After Newark offered Amazon $2 billion in tax incentives, former governor of New Jersey, Chris Christie, was so confident about the benefits of having HQ2 located in Newark that he promised Amazon an additional $5 billion. That $7 billion was larger than any tax break offered by the other 19 cities vying for Amazon's attention.

In order to understand the full meaning behind the potential Amazon might have brought to the citizens of Newark, it's instructive to consider the city's tumultuous political and economic history.

Newark, New Jersey, is one of the most populous cities in the U.S. and is one of the nation's major air, shipping, and rail hubs. Though several leading companies have their headquarters in Newark, including Prudential, PSEG, Panasonic, Audible.com, and IDT Energy, 31% of its residents live below the poverty line, and the city's unemployment rate is 12%. Not unrelated, it is the "most violent" city in New Jersey according to the FBI (Brown & Kiersz, 2018). The economic situation hasn't always been so bleak. Newark was once a flourishing industrial center. In the 19th century, it was known for its leather factories, breweries, and insurance industries. One historian noted, "its heavy industries, its whirring factories, its prosperous building trades, and its noted public works made it a confident and optimistic community" (Jackson, 1985, p. 275). He continues: "As late as 1927, a prominent businessman could boast":

Great is Newark's vitality. It is the red blood in its veins—this basic strength that is going to carry it over whatever hurdles it may encounter, enable it to recover from whatever losses it may suffer and battle its way to still higher achievement industrially and financially, making it eventually perhaps the greatest industrial center in the world (p. 275).

Soon thereafter, though, when the Great Depression hit in 1929, Newark suffered a precipitous decline in economic activity. Manufacturers and industrialists left the city and took their jobs with them. Conditions worsened when the Newark race riots broke out in 1967. Middle- and upper-class Whites fled the city, leaving behind poor and polarized communities of color. The demographics of the city have shifted since then, as the African-American population increased from 2.7% in 1990 to 52.4% in 2010, and the White population decreased from 97.2% to 26.3% over the same period of time.

The adversity Newark experienced over the last century may have played a role in it being identified among a list of 20 potential locations for Amazon's second headquarters. As primary stakeholders, the
citizens of Newark along with their homes and communities, stood to experience an unknown mix of positive and negative consequences as a result of the decision.

**Bridging In-School and Out-of-School Learning**

About a year after we implemented Phase 1 of the project, we returned to the school. We wanted to continue our efforts to develop knowledge-eliciting mathematical tasks, and the timing was right to leverage the Amazon issue as a contextual scaffold for linking students’ community- and school-based knowledge.

Reports on Amazon’s deliberations around HQ2 intermittently appeared in the news. However, these reports only offered the perspectives of politicians and the business community. Too few of them captured the perspectives of local residents. We hoped that through our conversations with students and their families, we would learn how they were feeling about the prospects of HQ2 being located in their community, which in turn would help us to leverage their out-of-school knowledge and experience to develop mathematical tasks that were meaningful to them. Accordingly, we took a *funds of knowledge* approach to acquiring this knowledge.

Luis Moll and his colleagues use the term “funds of knowledge” to refer to “historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being” (Moll, Amanti, Neff, & Gonzalez, 1992, p. 133). In their attempts to better understand the border region between Mexico and the United States, they visited homes, conducted observations, and implemented interviews. What they found were diverse forms of funds of knowledge that include knowledge related to farming, sales, construction, trade, auto repair, contemporary medicine, and household management. And what they learned was that households possess a wealth of cognitive and cultural knowledge that provide a counterpoint to deficit framings of marginalized students and that can be leveraged as resources for classroom instruction.

Teachers’ schedules are already overloaded, and conducting home visits and writing up the findings takes more hours than they have available. In fact, teachers who have expressed a desire to visit their students’ homes have cited a lack of time as the primary reason that they do not engage in these activities (Gonzalez et al., 1993). Furthermore, accumulating students’ funds of knowledge requires an already existing, trusting relationship between teachers and their students, making it particularly complex for new teachers (Moll et al., 1992). Indeed, accommodating these realities is one of the motivations for this project. Accordingly, when we wanted to assess what the students in our project and their families knew and felt about the prospect of Amazon’s second headquarters being built in their community, we took an approach that we felt would be much more feasible.

**The Amazon Problem**

In our first interaction with students around the Amazon problem, we presented the issue to them. We showed them what the situation was like for Amazon’s first headquarters in Seattle (see Figure 3).
We mentioned the prospects of 50,000 new jobs, the $100,000 salaries, and the infusion of funds for construction. We also mentioned the tax incentives and how they could result in reduced government spending on things like roads, schools, libraries, health care, and housing. Then we had a whole-class discussion in which we gathered students’ thoughts about the issue and what they imagined to be the benefits and drawbacks of having HQ2 located in Newark. We posed questions like:

- What changes do you think you might see in Newark and in your neighborhood?
- Will that make things better or worse for your community?
- Do you know anyone who might be interested in working there?
- How do you think local and family businesses will be affected?
- What do you think about the government’s decision to offer Amazon a $7-billion tax break?

Here are some of the things they said. We’ve italicized some of the words to forefront the mathematical ideas embedded in their responses – ideas about rate and change and so forth:

“Taxes will be raised and local business will suffer. If salaries increase, the area will become too expensive, causing business and families to be displaced,” said one student. “I live in downtown Newark and Amazon’s presence will drive housing demand so high that tenants may not be able to afford their monthly rent and other amenities,” added another.

Other students sounded more hopeful. They were eager to welcome Amazon to their town.

“There will be positive changes. Security presence will be high and that will reduce the crime rate in the city. Also the city’s economy will improve and that will put a favorable spin to our status. It will make things better and more people will come back to Newark.”

After this discussion with students, we asked them to use the same questions we had asked them in class to interview a parent, caretaker, or any other member of their community about the issue. This is the strategy we used to approximate a funds of knowledge, in-home visit.

Figure 3. Amazon’s first headquarters in Seattle, Washington
When students returned their completed interview protocols to us, we independently read their responses and identified the same three primary themes in the data: 1) space for Amazon, 2) housing and rental prices, and 3) traffic. Though students and their families generally expressed optimism about HQ2, an underlying concern was perceptible across their responses. They wondered, “Where would Amazon construct HQ2?” They were concerned about “the degree level necessary to fill the positions.” And they worried that “housing demands [could be] so high that tenants may not be able to afford their monthly rent and other amenities.” In response to what we had learned, we created the following three mathematical tasks, one for each of the themes we identified. Each of these tasks was designed to leverage the community knowledge we had assessed as a resource for students’ mathematical instruction.

*Where Will It Go?* is an activity connected to concepts of geometry and measurement. Students were given a live Google map of Newark (Figure 4) and asked to explore where Amazon could locate HQ2. We stated the problem as follows: Amazon’s Seattle campus is 8 million square feet. That’s equal to 2,828 feet on a side, or about 3,000 square feet. Look at the map of Newark and try to spot an empty place of this size where Amazon could build HQ2.

Students used the map’s *measure distance* feature to lay out an area approximately equal to the size of Amazon’s Seattle campus, and they considered whether it would be worthwhile to trade open spaces like parks and gardens. Finding a place for HQ2 proved to be a struggle. They soon realized that there was no viable place unless the city compromised an area currently occupied by places such as West Side Park (yellow marker in Figure 4), Fairmount Memorial Cemetery (red marker), the Prudential Center (blue marker), or the Red Bull Arena (green marker).
Home and Rental Prices was the second task we developed. We directed students to the website of an online real estate database company. There they found graphs of Newark rental prices and home values over a period of about the last ten years (see Figure 5). We had them analyze those graphs using questions like:

- What’s going on here?
- What do you notice about how these prices have been changing?
- When were home values increasing the fastest?
- What do you think was happening with rental prices between 2012 and 2014?

Then we had them make predictions about what the graphs would look like if Amazon were to locate HQ2 in Newark and provide real-world and mathematical justifications for those predictions.

Students used hand gestures, pointing upward, to denote the regions of the graphs where home and rental price were increasing. They gestured to indicate “more steeply” when predicting what the graphs would look like if Amazon were to locate HQ2 in Newark. When asked to justify their predictions, one student explained,

When Amazon comes in, a lot of jobs are gonna come in, as well, and a lot of people are gonna move in, and those people might have more money than the current residents of Newark. So current homeowners might sell their homes at higher prices to make profit off of, like, those influxes of people. So the home prices are definitely gonna go up.

In response, another student reiterated a concern that current residents might be displaced due to the rate hike.

For the third and final task, students participated in a NetLogo (Wilensky, 2009) participatory simulation called Gridlock (Figure 6) (Wilensky & Stroup, 1999). All the students connected to the simulation from Chromebooks and each student controlled one traffic light in a fictional town called Gridlock whose road design and traffic situation we used to simulate what traffic might look like in Newark if Amazon were
to locate there. Their task was to work together to find a way to improve traffic flow. In order to do that, they first had to mathematize what good traffic flow means. Three graphs were provided: the number of stopped cars at any moment, the average speed of cars, and the average wait time of cars. Students worked together to iteratively develop a strategy to optimize traffic flow by referencing the real-time data produced by the three graphs.

One strategy used by the students involved monitoring the traffic at their intersection and changing their light to maintain traffic flow. This proved to have little effect on traffic flow by any of the three measures represented by the graphs. Another strategy involved organizing the students and their traffic signals by rows in the grid and then changing their lights at regular intervals (e.g., every five seconds). That, too, had no significant impact on overall traffic flow. In the end they decided that, regardless of how they defined it, there were just too many cars moving through the grid. Traffic never moved smoothly. Hence, an optimal solution could not be found in some algorithm for coordinating the changing traffic lights; it had to be about reducing the number of cars. That realization was followed by a discussion in which students seemed at a loss about how to facilitate “good” traffic. They suggested that Newark would have to find a way to limit the number of cars that could flow through town at one time. Otherwise, given that severe traffic congestion was an inevitable consequence of the decision, it may not be worth locating HQ2 in Newark.

Discussion

We undertook the KEMA project to find ways to help teachers learn more about their students. We acknowledged that learning is more effective when teachers leverage this knowledge for their instruction, and more importantly, this knowledge is essential to building the kinds of caring relationships that are fundamental for classrooms to operate as communities for learning. The approach we took to assessing this knowledge was to develop tasks that—in concert with follow-up discourse moves that press students
to dig deeper—could reveal their home, community, and cultural knowledge. We know these tasks are hard to find and even harder to create.

We developed two kinds of tasks that we found to be productive. In our first phase of the project, we administered modified Would You Rather? tasks. We modified these tasks guided by the following principle: Tasks should compel students to make a choice and defend their decision. These tasks should feature both mathematical concepts and real-world contexts that students could conceivably relate to.

The tasks we implemented proved to be revealing of students’ mathematical knowledge while also providing a window into their lives at home and in their community. However, as revealing as they were, they didn’t generate the kind of whole-class discussion that would allow us to explore students’ responses more deeply or make connections across their responses. Thus, we couldn’t be sure that the individual responses we received were indications of some sort of shared community knowledge. We needed more evidence.

As a result, we took an approach in Phase 2 of the project that was guided by a different design principle: Identify a compelling local issue and leverage it as a contextual scaffold for bridging students’ out-of-school community knowledge with their in-school learning of mathematics. The issue we identified was the prospect of Amazon locating its second headquarters in Newark. This issue proved to be the kind of compelling, contextual scaffold we hoped it would, as the task bridged students’ in-school and at-home knowledge, and our implementation helped us learn quite a lot about them and their families. We were then able to use that knowledge as contexts for the development of three mathematical activities that students engaged with as if they were meaningful to them. That is, their mathematical engagement in rich tasks was structured by their own thoughts and concerns related to the contextual scaffold we had provided.

Interestingly, the day after the Amazon HQ2 decision announced its decision to locate in Long Island City, members of labor unions and progressive grassroots organizations gathered in Queens to state their opposition to a decision they believed would widen current income gaps and exacerbate the city’s ongoing displacement crisis. They spoke out against the $3 billion in tax breaks promised to Amazon by New York Governor Cuomo and New York City Mayor de Blasio amidst the city’s ongoing infrastructure funding needs. They also voiced their concerns about the deleterious effects Amazon might have on local businesses, new incentives to raise the costs of housing in an already competitive market, and the realization that public subsidies were given to Amazon whose new offices would be built on land that had been reserved for the construction of 1,500 units of affordable housing. We find it noteworthy that this range of reactions to Amazon’s decision resembled those that we heard from students and their families. They and other Newark residents may be breathing a sigh of relief that their city wasn’t chosen.

Towards a Responsive Pedagogy for Critical Mathematical Inquiry

Students’ multiple mathematical knowledge bases constitute the funds of knowledge upon which new knowledge is constructed. Consequently, eliciting this knowledge is fundamental to engaging students in critical mathematical inquiry (CMI). Indeed, critical consciousness, by definition (Freire, 1970), can only
develop from the awareness of one's own circumstances and reflection on one's own experiences. Thus, if teachers wish to operate their classrooms as participatory venues for CMI, it is essential that they assess their students' multiple mathematical knowledge bases. The two kinds of tasks we implemented were shown to do just that. We have taken this opportunity to share the design principles we identified with other teachers who wish to enact a responsive pedagogy. By engaging students in knowledge-eliciting mathematical activity, teachers can be better prepared to reveal their students' knowledge and connect it to the mathematics they intend to teach.

References


Debasmita Basu is a doctoral student in mathematics education at Montclair State University in northern New Jersey. As a high school mathematics teacher in India for four years, she was dismayed that her students tended to consider mathematics as a set of rules and formulas with little to no connection to their lives. Hence, with the greater goal of changing the nature of school mathematics, Debasmita started her doctoral studies in 2014. Her research agenda focuses on designing mathematical activities that aim to cultivate students’ critical consciousness towards various social and environmental justice issues and help them realize the power and value of mathematics.

Steven Greenstein is an associate professor in the Department of Mathematical Sciences at Montclair State University. He enjoys thinking about mathematical things—and how people think about mathematical things. Through his work, he aims to democratize access to authentic mathematical activity that honors the diversity of learners’ mathematical thinking, that is both nurturing of and nurtured by intellectual agency and that is guided by self-directed inquiry, mathematical play, and the having of wonderful ideas.