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Fractions, Decimals, and Percents : A Fifth Grade Curriculum Unit

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FRACTIONS, DECIMALS, AND PERCENTS: A FIFTH GRADE CURRICULUM UNIT

Teaching and Learning With Understanding

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Independent Study

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ABSTRACT

This independent study outlines a fifth grade curriculum unit for fractions, decimals, and percents. The unit utilizes a problem-solving approach to lead to deep understanding of fractions and their relations to decimals and percents.

The study is influenced by writings by Hiebert, Dewey, Piaget, Vygotsky, and Fosnot, and uses a lesson planning format created by Hal Melnick.

Four explorations drive the unit:

Exploration 1: What is a fraction?

Exploration 2: Going Deeper With Fractions

Exploration 3: Connecting Fractions to Decimals and Percents

Exploration 4: Comparing Fractional Amounts

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CHAPTER 1:

INTRODUCTION

CHAPTER 1: INTRODUCTION

Fractions, decimals, and percents. For many, the very mention of these terms evokes the memories of endless rules to be applied, endless numbers to be reduced, cross-multiplied, and factored. But ask a child what one-sixth times four-fifths actually *means*, and the question will more than likely be met with silence.

This independent study is focused on creating a unit of study exploring fractions, decimals, and percents -- a study that supports children in uncovering the *meaning* behind all those abstract symbols.

It is impossible to embark upon a study of mathematics without first considering how children learn. Will children learn if we simply model certain procedures for them and provide opportunities for practice? Or is there something else -- something more -- that we need to consider as we explore the terrain of mathematics with children?

The question is, of course, rhetorical. There *is* something more. For me, that "something more" is vitally linked to two things. The first is understanding. *Do the children get it?* How do we know? How do we teach so that they get it?

The second but no less significant aspect is joy. *Do the children love it?* How do we structure the learning situations so that the children approach mathematics with genuine joy?

STUDY OVERVIEW

Chapter 2 examines several theories about learning, both generally and specifically regarding mathematics. Chapter 2 also explores the notion of understanding. This chapter then delineates the characteristics of a classroom that emphasizes understanding and joy, and finally outlines the principles that guide curriculum design.

Chapter 3 introduces the "big ideas" to be addressed in the study, along with the New York State standards that are connected to them. This chapter also revisits the guiding principles is terms more specific to fractions, decimals, and percents.

Chapter 4 lays out the scope and sequence and a sample lesson plan.

Chapter 5 concludes the study and summarizes the key points put forth.

CHAPTER 1: INTRODUCTION

CONTEXT

I am currently in my second year of teaching third grade at P.S. 69 in the Soundview area of the South Bronx as a Teach For America corps member. However, this curriculum unit is designed for a 5th grade class, as I will be teaching 5th grade math at a new charter school for the 2005-2006 school year. The school is called KIPP: Always Mentally Prepared Academy, and is part of the nationwide network of charter schools in the Knowledge Is Power Program (KIPP). The school will eventually become a grades 5-8 middle school, but will only have 5th graders in the first academic year.

In this school, not only will there be a 90-minute block for mathematics for each student, but also an additional 75 minutes devoted to a class called "Thinking Skills." For the first 30 minutes of each day, students will complete a set of Thinking Skills problems. Then, during a 45-minute session later in the day, students will discuss these problems, going over each in detail, sharing alternative solving solutions, and proving their reasoning. I will be teaching both courses.

CHAPTER 2:

LEARNING MATHEMATICS WITH UNDERSTANDING

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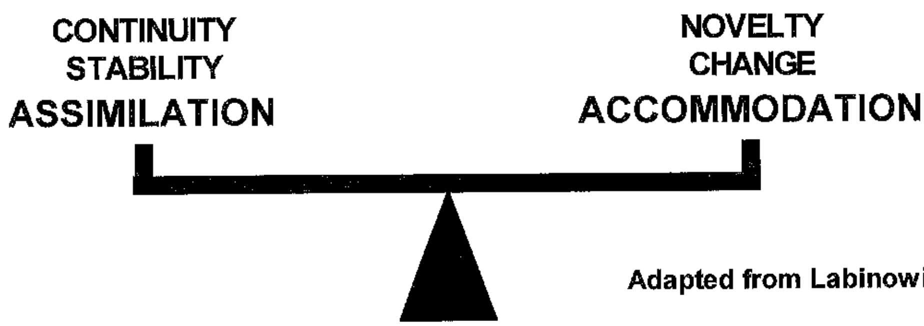
<u>The first question: How do children learn?</u>

The Swiss developmental psychologist Jean Piaget devoted his career to studying how children learn, and his views have had significant impact on educational theory. In Piaget's view, children have a constantly shifting mental framework -- a *schemata* -- for processing and organizing information and ideas. The dynamism of this schemata is fueled by two processes: assimilation and accommodation. Assimilation is the process of "incorporating our perceptions of new experiences into our existing framework" (Labinowicz 1980, p.36). Accommodation, on the other hand, is to "modify and enrich structures in our framework as a result of new input demanding changes" (Ibid, p.37). This balancing act between the two processes -- between stability and change, assimilation and accommodation -- can be portrayed thus:



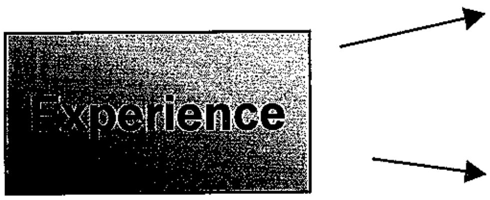
Adapted from Labinowicz, p.36-37

It may be helpful to think in more concrete terms. Labinowicz describes the hypothetical learning situation of a child encountering a squirrel for the first time (p.29-30). Already possessing the linguistic category "kitty" for four-legged furry animals, she at first *assimilates* the new information into her existing schemata and thinks the squirrel is a type of kitty. However, the child then sees the squirrel stand up on its hind legs, something she has never seen a squirrel do. Novelty. Change. Disequilibrium is created. She then *accommodates* the new information and creates a "funny kitty" category for kitties that stand on their hind legs. Later, her mother gives her the correct label, and a new category is created -- *squirrel*, creating equilibrium again. This constant balance of equilibration between internal mental schemata and experiences in the world is how Piaget characterized learning.

Knowledge, it follows, is NOT being absorbed passively from the environment, nor is it coming solely from a place within the child. The *interaction* between the child's mind and the environment is what creates learning. The child constructs his interpretation of the world through this constant interaction.

The scholar John Dewey also posited that children learn through interacting and making sense of the world around them. Learning occurs neither wholly from within or wholly from without -- it occurs in the interaction between self and world. Dewey called this process "experience." But more than Piaget, Dewey emphasized the importance of the *quality* of experience in learning. "Everything depends on the *quality* of the experience which is had" (Dewey 1938, p. 27). Dewey argued that the quality of any given experience should be judged by its effect on future experiences. The Piagetian assimilation and accommodation that occurs during any given experience, therefore, can only be deemed "educative" if that change in schemata allows the learner to continue to change in desirable ways. He writes:

Just as no man lives or dies to himself, so no experience lives and dies to itself. Wholly independent of desire or intent, every experience lives on in further experiences... Some experiences are mis-educative. Any experience is mis-educative that has the effect of arresting or distorting the growth of further experience. An experience may be such as to engender callousness; it may produce lack of sensitivity and responsiveness. Then the possibilities of having richer experience in the future are restricted. (Dewey 1938, p.25-27)



Educative -- present experiences live "fruitfully and creatively" (Dewey 1938, p. 28) in future experiences

Mis-educative -- present experiences narrow the field of future experience

<u>What are the implications of learning theory for math instruction?</u>

The implications for math instruction, then, are twofold. **First, we must strive to create a balance between equilibrium and disequilibrium, between assimilation and accommodation.** There must be activities that create stability and continuity for children -- they must be able to envelop <u>many</u> experiences into their *existing* schemata to create a sense of equilibration, reinforcing what exists in their minds. For example, it is helpful to have many experiences with unit fractions reinforce the concept that *as the denominator gets larger, the fraction gets smaller.* This could take the form of discussing and manipulating objects and pictures (fractions strips, candy bars, pizzas, cakes, etc.) on several occasions to create and strengthen the framework for that concept. However, we must also create experiences that foster *disequilibrium* in children's interaction with their environment.

Continuing the previous example, when the children are presented with fractions that are NOT unit fractions, their schemata is challenged with novelty. The child may know and conceptualize that one-third of a pizza is greater than one-eight of a pizza because the slices are bigger, but what happens when you are comparing one-third of a pizza with THREE-eighths of a pizza? The child's simple schemata of "as the denominator gets larger, the fraction gets smaller" is no longer sufficient to satisfactorily reach equilibrium with this novel experience. The process of *accommodation* is then catalyzed as children have continuing experiences with this same type of disequilibrium, ultimately resulting in the creation of the concepts *as the numerator gets larger, the fraction gets larger*, and *one must consider both the numerator and the denominator when comparing fractions* (among many other possibilities).

Second, we must select the kinds of experience that are educative, that live on "fruitfully and creatively" in future experience. Conversely, we must avoid designing experiences that close off children to continued learning. Going back to our fraction example from above, it may be easier to provide an experience for children that automaticizes their learning of the rule *as the denominator gets larger, the fraction gets smaller*. The teacher could simply present a series of unit fractions in abstract symbol form, teach the rule to the children without physical (manipulative or visual) representation, and give children several examples to which to apply the newly learned rule. It is virtually certain that most children will be able to complete that task with accuracy without difficulty -- since the only knowledge required is how to compare the magnitude of numbers -- and so the teacher may feel that "learning" has indeed occurred. However, no conceptual understanding of fractions has been developed. When those children are then presented with fractions that are NOT unit fractions, they have little basis on which to construct further meaning. They are reliant on the teacher for yet another abstract symbolic manipulation of numbers -- this time, probably some algorithm for converting fractions to common denominators and comparing the numerators -- that again provides them with little conceptual understanding. When asked to find a fraction of a number (say, one-fourth of twenty), these children are again at a loss, again relying on the teacher for a set of algorithms with little meaning, algorithms that are easily forgotten and rarely flexibly applied. Furthermore, the students fail to see the connections between all of these tasks, the realization of which ultimately leads to the understanding that fractions name part of a whole. The first experience has NOT lived on fruitfully or creatively in future experiences; it has, as Dewey warned against, "increased a person's automatic skill in a particular direction and yet [landed] him in a groove or rut" (Dewey 1938, p. 26). The child has had a "mis-educative" experience. Instead, we must strive to provide experiences that open the child's mind to future experiences.

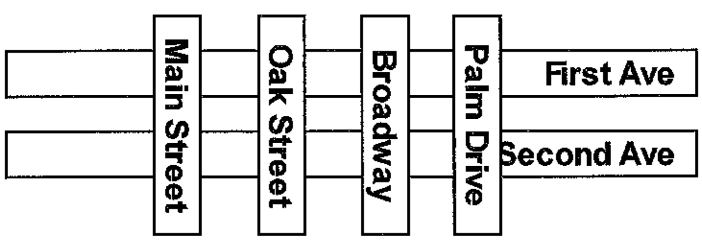
In sum, then, consideration of Piagetian and Deweyan theory yields two major implications:
1) We must strive for balance between assimilation and accommodation.
2) We must design experiences that are *educative* -- those that pave the way for future experiences.

What does understanding mean?

Do they get it? Most teachers ask themselves this question as they plan curriculum and engage with students. But "getting it" is an elusive concept to define. What does it mean to "get it?" One of the most useful frameworks for me is Skemp's instrumental understanding and instrumental understanding paradigm.

It can be understood thus: imagine two men living in the same city. One man is a work-a-holic, and spends all of his time at work or at home. Every day, he travels from home to work, and from work to home, using the same route every day. He is incredibly efficient at traveling this route. In his mind lives this understanding: *to get home, I need to turn right at Main Street, left at First Avenue, and right at Palm Drive.*

Contrast him with our second man. Our second man is a taxi driver. He rarely travels the same route every day. He picks up passengers and brings them to their destinations, using what he knows of the city's layout. He does not have a defined algorithm for any particular route, but he does use what he knows about the efficiency and relativity of various routes and applies that knowledge flexibly. In *his* mind lives this understanding:



Our first man can be said to have what Skemp calls "instrumental understanding" -- knowing what to do and how to do it [my paraphrasing]. He knows how to get from home to work, and from work to home. Our second man, the taxi driver, possesses "relational understanding" -- knowing what to do and *why*. His mental map of the city does not make him an expert at navigating every route, but it does allow him to flexibly and comfortably arrive at virtually any destination.

Finally, consider this task presented to each man: what is the best way to get from the park to the zoo?

It is obvious which man will be able to complete the task. The taxi driver understands how the parts of the city are related to each other, and thus he is more able to tackle any routing question easily. The work-a-holic is extremely adept in a very narrow situation, but his lack of relational understanding limits his *fluency* to one area only.

The National Research Council espouses a similar definition of understanding, which they dub "conceptual understanding" (NRC 2001, p. 118):

> *Conceptual understanding* refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. (NRC 2001, p. 118)

Let us take an example in mathematics. Imagine two teachers, Mr. I and Mr. R. Mr. I teaches for instrumental understanding -- he wants his students to know what to do and how to do it, with efficiency and accuracy. Mr. R is also concerned with efficiency and accuracy, but he wants his students to have relational (or conceptual) understanding, and organizes his teaching to support this development.

Now consider this problem:

> Jack had 7 pencils. Tyrone had 4 pencils. How many pencils do they have in all?

Mr. I takes this approach: he knows that these types of questions are incredibly important in first and second grade mathematics, and that he will be evaluated on whether his students can answer this correctly. His approach is to teach a keyword strategy -- *the words "in all" tell you that you should add*. He knows that few students will have difficulty internalizing this rule. This approach will work with problems that are structured like this (asking students to sum two addends in a single-step algorithm) and is even flexible enough when the number of addends increases (for example, four kids with four quantities of pencils). Mr. I teaches this rule, and provides several opportunities for practice. His students score well on an assessment dominated by these types of problems.

Mr. R takes a different approach. He is aware of the need for his students to be proficient at these types of problems, but he also fits that into his larger goal of having his students become flexible problem-solvers. His approach is to emphasize, through extensive discussion, *why* these numbers should be added. He wants his students to come to this understanding: *addition is used when quantities are being put together.* Over the unit, he exposes his students to both addition *and* subtraction problems of this sort, so that students can see that *subtraction is used when quantities are being taken away*, and so children can see the inverse relationship between addition and subtraction. Mr. R always insists that students articulate *why* numbers are added in this type of problem. In this way, Mr. R's students are not only prepared to tackle problems of the sort above, but also many other problems.

The contrast between the approaches may not be clear initially; in both classes, students are able to accurately and efficiently complete simple one-step addition problems. However, the difference between Mr. I's and Mr. R's teaching becomes stark when the students are challenged to solve different types of problems.

Consider this next type of problem, which teachers may use as an entrée into multiplication:

Jack and Tyrone each have several boxes of pencils. Jack has 4 boxes, and each box has 6 pencils. Tyrone has 3 boxes, and each box has 5 pencils. How many pencils do they have in all?

Mr. I's students, having had much success with the "in all" rule, apply their understanding instrumentally -- *when you see the words "in all," add the numbers up.* Many of them add all the numbers together and get the answer of 18. When told that "in all" can also mean multiplication, and that they are now supposed to multiply 4 and 6, and then add that to the sum of 3 x 5, they are confused. The next time they encounter a word problem with "in all," their confidence is shaken. It is unclear to them which operation to choose.

Mr. R's students, having understood the *concept* of addition, are initially not quite sure what to do. They know they are putting numbers together, but most of them are probably more thoughtful about what numbers they need to put together. The subsequent discussion, then, may focus on adding four 6's and three 5's, providing a powerful segue into discussing the concept of multiplication.

In Deweyan terms, the contrast between these two types of experiences is significant.
Mr. I's students' experience has produced some level of learning, but the experience is not necessarily "educative." Dewey posits that "every experience enacted and undergone modifies the one who acts and undergoes, while this modification affects, whether we wish it to or not, the quality of subsequent experiences" (Dewey 1938, p. 35. In judging the quality of any experience, then, Dewey prods us to ask:

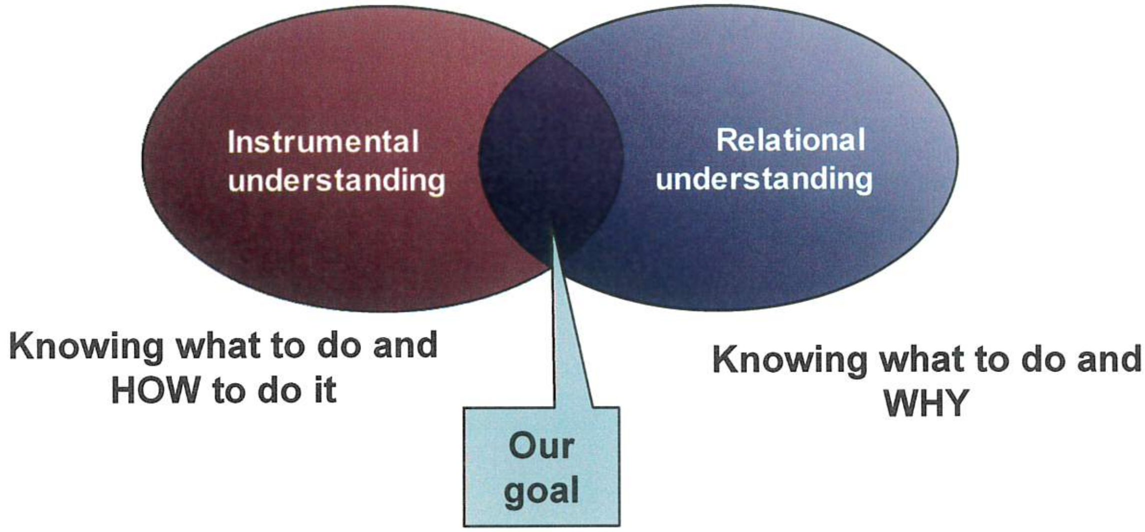
Does this form of growth create conditions for further growth, or does it set up conditions that shut off the person who has grown in this direction from the occasions, stimuli, and opportunities for continuing growth in new directions? (Dewey 1938, p. 36)

In the more concrete context of our problem above, the *instrumental* understanding in Mr. I's class shut off the children to opportunities to discuss the concept behind the operation called "addition," making it difficult for them to extend their expertise into other areas.

Those in Mr. R's class, by contrast, having developed *relational* understanding of addition, are primed to develop further in subsequent experiences.

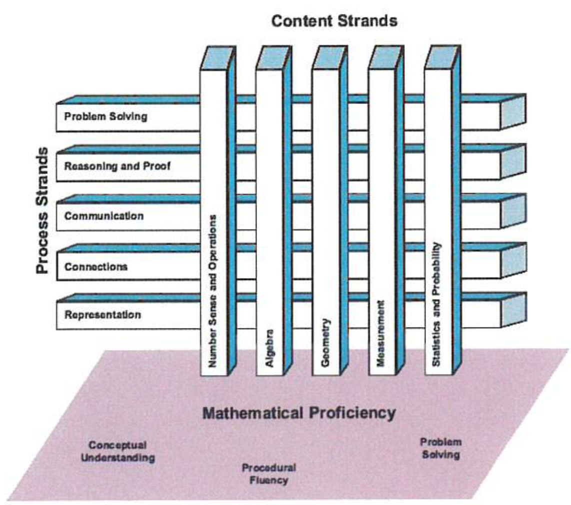
This is not to say that instrumental understanding is not useful as well. A severe lack of instrumental understanding significantly hampers efficacy -- knowledge of the city's layout does a taxi driver no good if he does not know how to drive a car. Mr. R's students still need instrumental understanding -- they still need to know *how* to add numbers.

This brings us to our third implication for math instruction -- **our teaching must strive for both instrumental understanding *and* relational understanding.** The figure on the next page summarizes this implication:

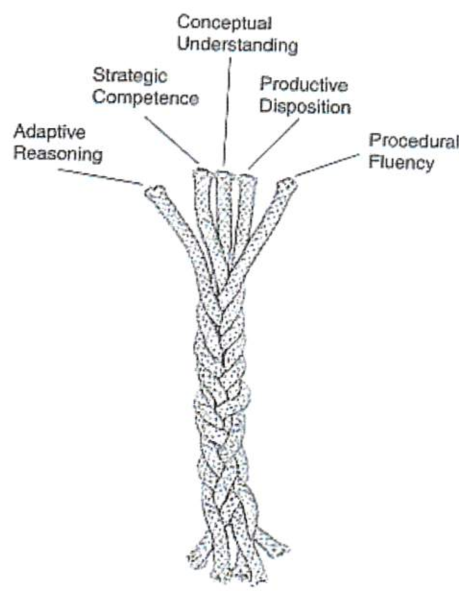


The major bodies that govern curriculum development in mathematics are clearly aligned on this point. *Conceptual (relational) understanding is important.* Both the National Research Council and New York State emphasize conceptual understanding as part of what it means to be "mathematically proficient." They also include several other strands:

New York State Mathematics Standard 3



Adapted from *Mathematics Framework for the 1996, 2000, and 2003 National Assessment of Educational Progress.*



Intertwined Strands of Proficiency

Source: New York State Standards Grades 3-8

Source: NRC 2001, p. 5

These two frameworks are quite similar. The NRC's structure is perhaps more detailed, breaking what New York State calls "problem-solving" into the two strands of "strategic competence" and "adaptive reasoning." For me, this difference is not incredibly substantive -- these two strands are inextricably linked. To have strategic competence, one needs to be able to reason adaptively.

For me, the more important difference between the two frameworks is the NRC's inclusion of the strand "productive disposition," which the NRC defines as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC 2001, p. 5). Put more simply, it is the **joy factor**. *Do the kids love it?* The joy factor, I believe, is the engine behind all of the other strands.

The joy factor

This part is simple. If there is no joy, it is difficult to learn, let alone learn with understanding. Focus is lost, motivation is weak, and the perseverance to productively deal with uncertainty -- the novelty that creates the accommodation Piaget describes -- is non-existent.

Even Dewey acknowledges the importance of joy: "The quality of any experience has two aspects. [First], there is an immediate aspect of agreeableness or disagreeableness..." (Dewey 1938, p. 27).

But what does this look like in the classroom that emphasizes understanding? It is not sufficient to make class "fun." There are a plethora of examples of "fun" activities that serve little mathematical or developmental purpose.

More lasting -- and therefore more educative in the Deweyan sense -- is helping children feel *successful* at mathematics. It is no secret that children love things they are good at. Designing experiences in which they can feel successful -- and still develop *relational* understanding -- is therefore crucial. In this way, the joy factor is intimately linked with Vygotsky's notion of a *zone of proximal development*. The experience must not be too easy such that little learning occurs, but it must not be too difficult such that frustration sets in.

Every experience, then, must have an element of comfort for the children. From this level of comfort, the children can then spring to understanding of new content. We must, however, have a *creative* sense of what that comfort level might be. The comfort need not be derived only from familiar content. It can be a story, an analogy, a mnemonic device, a chant set to the rhythms with which the children are familiar. When the children are enjoying themselves, their focus -- and ability to tackle challenging content -- is maximized. The teacher must take care, however, not to rely too heavily on methods that develop only instrumental understanding. This type of teaching would simply take "traditional" teaching --

algorithms to be memorized -- and make it fun. The teacher still needs to consider *relational* understanding in designing classroom tasks.

But the bottom line is this: the kids have to love it. Like Shel Silverstein's pet rhinoceros in *Who Wants A Cheap Rhinoceros*, no matter how much grief it causes us, math is just so easy to love.

<u>What does a classroom that teaches for understanding look like?</u>

Hiebert (1997) outlines the major dimensions of a classroom that emphasizes understanding:

DIMENSIONS	CORE FEATURES
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave behind something of mathematical value
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose--to solve problems Used for recording, communicating, and thinking
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes

1–1 Summary of dimensions and core features of classrooms that promote understanding

All of these dimensions are crucial, but the social culture is particularly important. Hiebert advises teachers to create a culture where discourse is primarily about methods and where reasoning is the guiding principle of class debate. I could not agree more. In my third grade classroom, students were always expected to justify their methods with mathematical reasoning and proof, and that consistent practice enabled them to reach a deeper level of mathematical understanding. In my classroom, every student had the opportunity to be a "teacher." Students would go up to the overhead and justify their reasoning and explain their methods. Other students would be encouraged to ask "why" questions, so much so that nearly every statement the "teacher" made was met with a "why" question. Afterwards, students engage in evaluation -- giving feedback on what "teachers" did well and what they could work on. Hiebert considers this process -- reflection and communication -- vital to creating understanding:

Reflection occurs when you consciously think about your experiences... stopping to think carefully about things, to reflect, is almost sure to result in establishing new relationships and checking new ones. It is almost sure to increase understanding.

Communication involves talking, listening, writing, demonstrating, watching, and so on. It means participating in social interaction, sharing thoughts with others and listening to others share their ideas... often we can accomplish more than if we worked alone. Furthermore, communication allows us to challenge each other's ideas and ask for clarification and further explanation. This encourages us to think more deeply about our own ideas in order to describe them more clearly or to explain or justify them. (Hiebert 1997, p. 5-6)

We must strive to create classrooms in which reflection and communication are a natural and integral part of what "mathematics" is. If a student in my class is asked, "What do you do in math?" I would be pleased if the response is, "We talk about our thinking."

Guiding principles

My goal is to teach for understanding, and for my students to learn with understanding. Examining learning theory, exploring understanding, considering the joy factor, and laying out what a classroom that emphasizes understanding looks like yields several principles that guide this study:

1) We must strive for balance between **assimilation and accommodation**.

2) We must design **experiences that are *educative*** -- those that pave the way for future experiences.

3) We must strive for both **instrumental understanding *and* relational understanding.**

4) We must ensure that our students approach mathematics with **joy**.

5) We must create a social culture in which students **communicate and reflect upon their thinking and reasoning**.

If I can adhere to the above principles, it is my belief that the answers to my two primary questions -- *Do they get it?* And *Do they love it?* -- are two resounding yes's.

CHAPTER 3:

FRACTIONS, DECIMALS, AND PERCENTS: BIG IDEAS AND STANDARDS

CHAPTER 3: BIG IDEAS AND STANDARDS

Fosnot and Dolk (2002) use the framework of "big ideas" to guide curriculum development in different areas. These big ideas are the fundamental mathematical relationships underlying different content strands, and are the realizations that teachers hope their students construct throughout the unit. It is useful to consider the big ideas in relation to the new (revised) content standards put forth by New York State:

Big Ideas (Adapted from Fosnot and Dolk, 2002)	**New York State Standards (Gr. 5)** *Students will be able to…*
Fractions are relationships of parts to wholes. Fractions are connected to multiplication and division.	**5.N.20**: Convert improper fractions to mixed numbers, and mixed numbers to improper fractions
Any fraction can be renamed – fractions with different denominators that name the same amount are equivalent.	**5.N.4**: Create equivalent fractions, given a fraction **5.N.19**: Simplify fractions to lowest terms
Fractions are more easily compared if they have like denominators. To compare two fractions, the whole must be the same.	**5.N.9**: Compare fractions using <, >, or = **5.N.5**: Compare and order fractions including unlike denominators (with and without the use of a number line) *Note: Commonly used fractions such as those that might be indicated on ruler, etc.*
Adding and subtracting fractions is easier if the denominators are the same.	**5.N.21**: Use a variety of strategies to add and subtract fractions with like denominators **5.N.22** : Add and subtract mixed numbers with like denominators
Percents are fractions based on a 100-part whole.	**5.N.11**: Understand that percent means part of 100, and write percents as fractions and decimals
Decimals are fractions using base-ten equivalents and place value.	**5.N.8**: Read, write, and order decimals to thousandths **5.N.10**: Compare decimals using <, >, or = **5.N.23** : Use a variety of strategies to add, subtract, multiply, and divide decimals to thousandths

In designing the curriculum unit, then, I need to consider both the big ideas I hope to help learners construct as well as the standards learners need to meet by year's end. These two *must* be considered together, as it is difficult to come to a *relational* proficiency in meeting standards without development of big ideas.

Going back to the guiding principles of the study yields more specific implications for the design of the unit:

1) <u>We must strive for balance between **assimilation and accommodation**</u>.

Though the big ideas do not necessarily develop in a linear sequence, it is important to ensure that some assimilation occurs before additional accommodation is demanded. For example, students will have incredible difficulty accommodating for the big idea *to compare two fractions, the whole must be the same* if they have not yet understood that *fractions are relationships of parts to wholes*. We must be sure to provide enough experiences for students to assimilate this crucial big idea.

By the same token, if we do not introduce novelty -- and thus spur the need for accommodation -- some big ideas will never develop. For example, we may provide students with many opportunities to assimilate the rule *if the denominator gets larger, the fraction gets smaller* with tasks involving comparisons of unit fractions only, as well as *as the numerator gets larger, the fraction gets* larger for fractions with like denominators only. However, if we do not challenge students to compare fractions with unlike denominators *and* unlike numerators, they will likely never develop the big ideas *fractions are more easily compared if they have like denominators* and *to compare two fractions, the whole must be the same*. Balance is key.

2) <u>We must design **experiences that are *educative*** -- those that pave the way for future experiences</u>.

One of KIPP's mottos is "No Shortcuts." It is an apt saying to apply here. For example, it may be simplest to present decimals in relation to money, since students tend to have familiarity with the dollars-and-cents decimal notation. However, we must make sure that students do not simply think of decimals as money and shut their thinking down. We must push for the understanding that each digit after a decimal point has a place value, so that students can be successful with *future* experiences that involve decimals beyond hundredths or those limited to tenths. As Dewey argues, the experiences we create must be educative -- they must live on "fruitfully and creatively" in future experiences.

3) We must strive for both **instrumental understanding *and* relational understanding**.

For example, we must make sure that students not only understand HOW to create equivalent fractions, but also WHY those algorithms work. We must also be sure that students are competent and efficient at useful algorithms (for example, finding the least common denominator of two fractions). We do not want our students needlessly recreating inefficient methods to compare two fractions once they have grasped the relational understanding underlying the algorithm.

4) We must ensure that our students approach mathematics with **joy**.

Many students approach fractions with dread. No wonder; since the subject is often taught abstractly, there is little concrete material to grasp onto as they navigate the minefields of endless algorithms to memorize. Instead, we must first make sure that any discussion of fractions is grounded in concrete terms, in contexts with which they are familiar. This can take the form of stories, pictures, demonstrations -- anything to help students who are making the transition from Piaget's concrete operational to formal operational stage.

Of course, the goofier the context, the better. As KIPP Bronx math teacher Frank Corcoran remarks, "The [goofy] stories serve a purpose. They provide a paradigm for struggling students to follow to a comfort zone in the classroom, and they entertain the more advanced students who otherwise might not pay attention to the details" (KIPP Math curriculum guide).

5) We must create a social culture in which students **communicate and reflect upon their thinking and reasoning**.

This needs little elaboration. I believe that students will learn most with *extensive* class discussions about fractions, methods, errors, etc. It is the teacher's duty to provide just enough guidance to prod the discussion in the right direction, but not so much that students abandon their own reasoning and rely on the teacher's thinking.

CHAPTER 4:

CURRICULUM UNIT: FRACTIONS, DECIMALS, AND PERCENTS

INTRODUCTION

This curriculum unit of fractions, percents, and decimals aims to meet the New York State performance standards outlined in Chapter 3, all the while creating *relational* understanding of the concepts explored. Therefore, instead of being taught in isolation, students are constantly asked to make connections between fractions, percents, and decimals as different ways of naming parts of a whole. One of the key themes that runs through the 5th grade curriculum in these areas is the ability to *compare* different amounts. Thus, the culminating exploration of the unit is concerned with comparing fractional amounts in authentic problem-solving contexts.

Another key theme that will be brought out in this unit is the usefulness of fractions, decimals, and percents in real-life contexts. Therefore, throughout the unit, students will encounter problems grounded in real-life context that push them to apply the concepts they have learned.

A third key theme is the use of models for fractions. A fraction, by itself, means little to the average 5th grader. We must help them construct visual and manipulative representations of each fraction to help them with the abstraction inherent in our symbol system. Therefore, students will constantly be asked to represent fractions with various models -- fractions strips, fraction towers, pattern blocks, clock faces, grids, etc.

A fourth theme concerns the sequencing of individual lessons. Often, lessons are taught in this way: a rule is given, and then students practice that rule on several examples until they master it. However, I do not believe that many students actually come to *relational* understanding in this way. Therefore, in this unit, the general model is to first push students to prove certain assertions using the visual models with which they are familiar, and only *then* to discuss the rule that can be implied from *their own reasoning*. In this way, we can create both instrumental *and* relational understanding. I believe that this simple change in sequencing can make significant differences in mathematical understanding and reasoning skills.

The unit is divided into four interrelated, carefully sequenced explorations. The **first exploration** is a foundational exploration, giving students ample experience with models to help them conceptualize fractions. The **second exploration** goes deeper with fractions, working with equivalence and simple operations. The **third exploration** connects fractions to decimals and percents. The **fourth and final exploration** utilizes three powerful problem-solving contexts to not only consolidate prior knowledge from previous explorations (assimilation) but also introduce the difficult task of comparing fractions (accommodation). In

INTRODUCTION

this way, the explorations are not isolated lessons but interconnected and build upon one another, such that older knowledge is constantly spiraled and revisited, and sometimes revised.

This unit attempts to meet the 5 implications drawn out in Chapter 2:

Balance between assimilation and accommodation. Ample time is give for each lesson such that students can assimilate important information before moving on to accommodate new information. For example, 3-4 sessions are allotted Exploration I: What is a fraction? because of the importance of assimilating visual models for abstract fraction symbols.

Educative experiences. Each lesson builds upon the last. For example, the visual models used in Exploration I are constantly referred back to and used in subsequent lessons as experiences to be built upon. Therefore, each lesson is designed to be useful not only in meeting that particular lesson's objective, but is designed to be useful in future lessons as well.

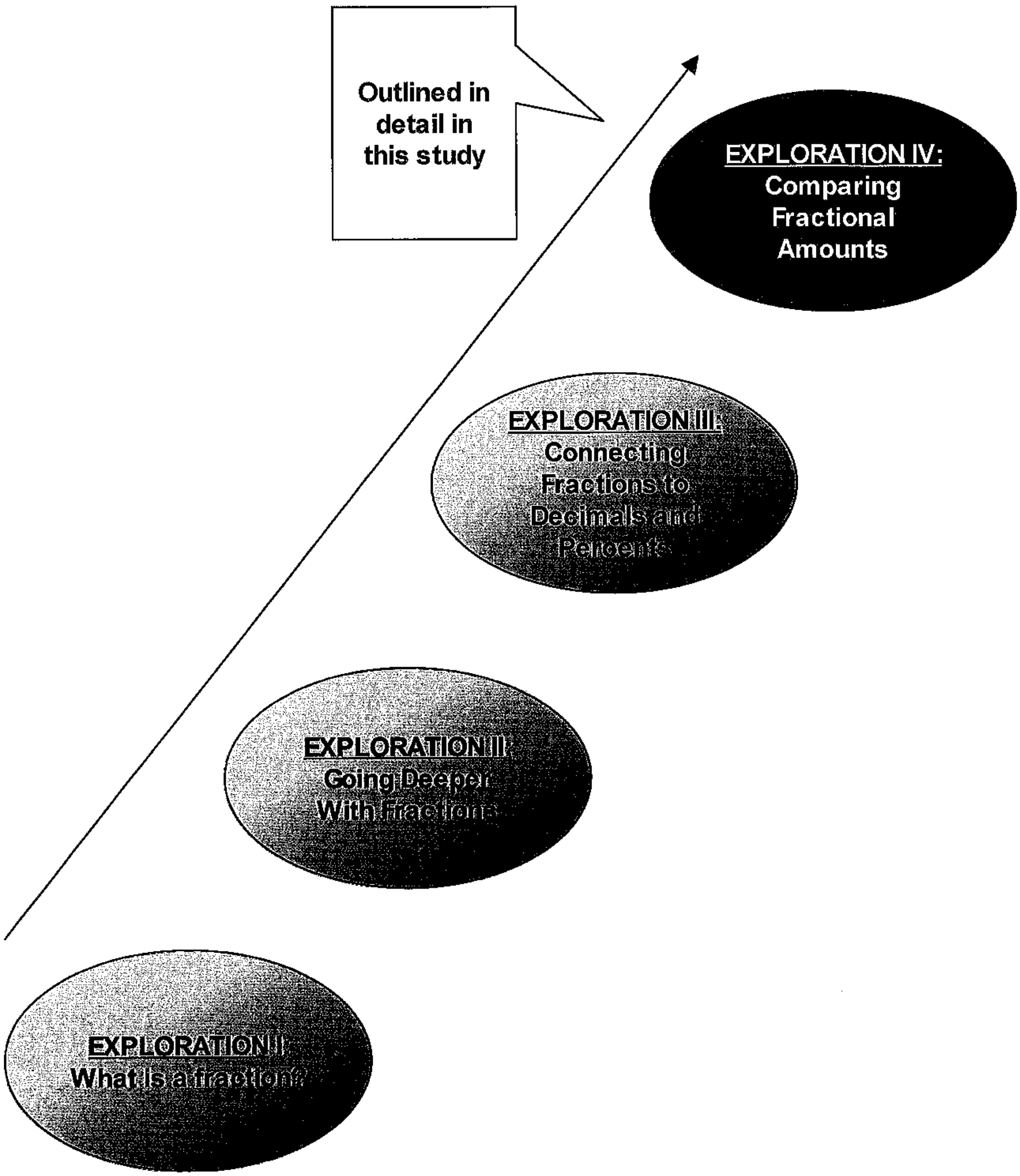
Both relational and instrumental understanding. The sequence of individual lessons -- model first, then generalized rule -- contributes to the development of both relational and instrumental understanding.

Joy. "Goofy" stories are incorporated to help students remain engaged and focused. In addition, the visual models are often created by the students themselves, which is interesting to them (art) as well as creates a sense of ownership.

Communication and reflection. Students will constantly be asked to prove their assertions with reasoning and logic, both verbal and visual. An important part of every lesson is the math congress discourse.

This section is set up as follows:
- Unit overview -- four explorations
- Exploration IV outlined
- The drinks problem in Exploration IV in detail -- Melnick lesson plan
- Materials and resources

UNIT CONSISTS OF 4 INTERRELATED EXPLORATIONS



EXPLORATION I AND II - THE FOUNDATION

EXPLORATION I: What is a fraction?

Sessions: 4-5
Students are introduced to various models to represent fractions -- fraction circles, fraction towers, pattern blocks, clock faces, grids. Students work to identify fractions of area, fractions of volume, fractions of a set, and fractions of numbers. The key big idea to be developed here is "a fraction represents a part of a whole, measured in equal parts."

BIG IDEAS	STANDARDS
Fractions are relationships of parts to wholes. Fractions are connected to multiplication and division.	

EXPLORATION II: Going Deeper With Fractions

Sessions: 4-5
Students work with fractions in more depth, encountering problems that challenge them to create equivalent fractions (including simplifying, work with mixed numbers and improper fractions, and adding and subtracting fractions with like denominators.

BIG IDEAS	STANDARDS
Fractions are relationships of parts to wholes. Fractions are connected to multiplication and division. Any fraction can be renamed -- fractions with different denominators that name the same amount are equivalent. Adding and subtracting fractions is easier if the denominators are the same.	5.N.4: Create equivalent fractions, given a fraction 5.N.19: Simplify fractions to lowest terms 5.N.20: Convert improper fractions to mixed numbers, and mixed numbers to improper fractions 5.N.21: Use a variety of strategies to add and subtract fractions with like denominators 5.N.22 : Add and subtract mixed numbers with like denominators

EXPLORATIONS III AND IV -- CONNECTIONS

EXPLORATION III: Connecting Percents and Decimals To Fractions

Sessions: 4-5

Students are introduced to the two other conventions of naming parts of a whole -- decimals and fractions. Students explore percents on hundred grids, and connect these amounts to fractions, including fractions that do NOT have a denominator of 100. Decimals are connected to students' knowledge of base-10 place value, as well as familiarity with the monetary system. Special attention is also given to ordering and comparing decimal amounts, as well as applying number operations (addition, subtraction, multiplication, and division) to decimal amounts.

BIG IDEAS	STANDARDS
Percents are fractions based on a 100-part whole. Decimals are fractions using base-ten equivalents and place value.	<u>5.N.11</u>: Understand that percent means part of 100, and write percents as fractions and decimals <u>5.N.8</u>: Read, write, and order decimals to thousandths <u>5.N.10</u>: Compare decimals using <, >, or = <u>5.N.23</u> : Use a variety of strategies to add, subtract, multiply, and divide decimals to thousandths

EXPLORATION IV: Comparing Fractional Amounts in Problem-Solving

Sessions: 4-5

In the culminating exploration, students are expected to apply all of the learning gained in previous explorations to solve two authentic problems in small groups. The first problem is the store problem, where students determine which store has the better deal on Gatorade. The second problem is the sandwich problem, where students determine which group of students received the most food. Both problems are solved in small groups and presented/discussed in a Math Congress. The final task is for *individuals* to prove, using as many models as possible, which fraction is greater -- 2/3 or 3/5. Students are expected to use a wide array of proofs here, including visual models, ratio charts, words, percents, decimals, and conversion using equivalent fractions.

BIG IDEAS	STANDARDS
Fractions are connected to multiplication and division. Fractions are more easily compared if they have like denominators. To compare two fractions, the whole must be the same.	<u>5.N.9</u>: Compare fractions using <, >, or = <u>5.N.5</u>: Compare and order fractions including unlike denominators (with and without the use of a number line) *Note: Commonly used fractions such as those that might be indicated on ruler, measuring cup, etc.*

EXPLORATION IV ORGANIZED AROUND THREE RELATED PROBLEMS

Our class decides to go on a picnic to Prospect Park. To prepare for this picnic, we need drinks, sandwiches, and candy bars for dessert.

DRINKS

We go to the grocery store, and see 18-packs of Gatorade selling for $16. Not wanting to be ripped off, we go to the corner deli and see 15-packs of Gatorade selling for $14. Which store has the better deal on Gatorade?

GROUP

SANDWICHES

When we get to the park, we split up into 4 groups, who sit at four different-colored tables. There are only 17 sandwiches to go around, and we split them up like this:

- **At the blue table, 4 kids get 3 sandwiches**
- **At the red table, 5 kids get 4 sandwiches**
- **At the green table, 8 kids get 7 sandwiches**
- **At the yellow table, 5 kids get 3 sandwiches**

GROUP

But some kids start to complain that this isn't fair. Do you agree? Why or why not?

CANDY BARS

Then, for dessert, we have a whole bunch of Snickers bars. But we don't have an organized way to distribute them. Two different groups of kids approach you. One group wants you to be part of 3 kids splitting 2 Snickers bars. The other group wants you to be part of 5 kids splitting 3 Snickers bars. Which group would you rather be in?

INIDIVIDUAL

NOTE: The drinks problem and the sandwiches problem are adapted from the cat food problem and sandwiches problem in Fosnot and Dolk (2002)

LESSON PLAN: THE DRINKS PROBLEM (I)

DRINKS

We go to the grocery store, and see 18-packs of Gatorade selling for $16. Not wanting to be ripped off, we go to the corner deli and see 15-packs of Gatorade selling for $14. Which store has the better deal on Gatorade?

GROUP

What is the mathematics in this lesson?

This lesson is centered around the idea that fractions are connected to division, as well as the fact that fractions can be expressed using decimals -- specifically, monetary amounts. Also, students will develop the idea that to compare fractional amounts, some common context must be arrived at -- for example, that we must compare the price of ONE Gatorade (or THIRTY Gatorades) to get a fair and accurate comparison. In addition to these content topics, the students will be utilizing a number of problem-solving skills, including framing the problem, communicating with others using a variety of media (verbal, written, visuals), and defending their findings.

Where does the lesson fall in the unit and why?

This lesson is the first problem in the final unit. It is a relatively straightforward problem -- only two sets of variables to be considered, a simple two-way comparison -- that draws upon all of the knowledge and skills developed in previous explorations in this unit. By contrast, the sandwiches problem is more complex -- four sets of variables to consider -- which is why it comes *after* the drinks problem.

NOTE: This lesson plan format was developed by Hal Melnick, Bank Street College of Education

LESSON PLAN: THE DRINKS PROBLEM (II)

What are your math goals for the students as they do this lesson?

Students will utilize several problem-solving strategies to approach this problem, including seeing fractions as division, using ratio tables, and finding a common base amount to create a fair comparison. Every team MUST come up with at least TWO ways to justify their answer. In addition, it is my hope that the ending whole-group Math Congress will help children see that there are a myriad of ways to approach the problem, and that their classmates have much to teach them.

What prior knowledge do you anticipate that the students bring (or should bring) to this lesson?

Students will bring to the lesson the content and skills developed in previous explorations, including:

- Fractions are connected to division
- Fractions can be renamed without changing the amount -- the concept of equivalence
- The ability to approach problems with a number of strategies, including drawing pictures, making tables, verbal descriptions, etc.
- The understanding that there is often more than one way to solve a problem

What materials and tools are needed for this lesson?

- 33 Gatorade bottles (no problem since I drink a lot of Gatorade)
- Chart paper
- Markers

How will the lesson unfold?

This lesson will occur over three sessions:

- The **first session** is the set up and exploration of the problem's parameters. Groups are set up, group norms are established and modeled, the problem is presented, and children have a chance to mentally chew on the problem with their partners (groups of three). I chose a group of three because the group is

LESSON PLAN: THE DRINKS PROBLEM (III)

small enough that each child has to be an active participator, and big enough that each child can occasionally sit back and observe the interaction between the other two children. The children will be grouped heterogeneously. At the end of the first session, the first Math Congress is held, primarily to discuss methods and approaches. It is crucial that the first Math Congress is held *before* any group is able to come to a correct answer. This will allow children to hear others' approaches before seeing a final answer (which may shut down their own thinking processes).

• The **second session** is the meat of the exploration. Children are given a large block of time to confer with teammates, use models and manipulatives if needed, and begin to create their solutions to the problems. A Math Congress will be held mid-way through the second session. This Congress is NOT about solutions, but about how best to *express* solutions. What is the best way to show your thinking? In the second half of the session, (after the Congress), the students work to create their poster -- their defense of their solution.

• The **third session** is about sharing solutions and methods -- sharing their posters. Each team will have a chance to share out with the whole group. An evaluation will occur after each share-out -- what did you find interesting or clever about this group's method? What did you find difficult to understand? How could they present their thinking better? It is important for the teacher give some guidance to the discussion, so that the big ideas (mentioned above and below) are emphasized.

What are some questions you could ask during the launch, exploration or sharing time?

The questions serve different purposes at different times. The questions at launch serve to activate prior knowledge, as well as highlight the importance of using the same "base" amount to compare. The questions during exploration serve to push thinking beyond one solution method and get struggling groups over the hump. The questions at sharing time serve to consolidate learning and help children connect to the underlying big ideas.

LESSON PLAN: THE DRINKS PROBLEM (IV)

LAUNCH:

• The deli has Gatorade for $14. The supermarket has Gatorade for $16. The deli is definitely cheaper, right?

• The supermarket has 18-packs of Gatorades, and the deli has only 15 packs. They cost about the same, so the supermarket has the better deal, right?

• How can we compare the prices fairly? (This is a thinking question that is not discussed, but presented right at launch to spur thinking).

EXPLORATION:

• Are there other ways of proving this? (for early finishers stuck on only one solution method)

• What if you wanted to buy just one Gatorade? Say the owner would sell it to you for the same price as the 18-packs or 15-packs. How much would the owners charge you at the supermarket? At the deli? (for those who are stuck)

SHARING:

• There were lots of different ways to solve this problem, but what did each group have in common? (pushing the group toward the big idea that a "base" amount is needed to compare fractions) -- this question is CRUCIAL

• Which way is the best way and why? (A wink wink question -- one with no one absolute right answer)

What do you expect student work to look like and sound like?

I expect student work to neatly and concisely express their solution method. I will focus solution explanations on agreeing on the "base" amount. Children's thinking, however, is messier than this, and so the posters will require planning and drafts. Ample time will be allotted for this process -- the neat articulation of often "messy" thinking.

I also expect students to debate the merits of the case using mathematical arguments developed in previous explorations.

LESSON PLAN: THE DRINKS PROBLEM (V)

What confusions might students have?

One key confusion could be that some children will divide the inverse way -- instead of $14 divided by 15 Gatorades, they will do 15 Gatorades divided by $14, yielding an answer that has units of Gatorades per dollar instead of dollars per Gatorade.

Another confusion is that some groups may have difficulty approaching the problem at all. Then, the second question under the exploration section above could prove useful.

How do you anticipate you will be ready to help children work through their confusions?

See modifications below. Also, for the inverse division confusion, push students to define the units their fraction will be in (Gatorades per dollar or dollars per Gatorade?)

What extensions (for more able students) and modifications (for differently able students) will you be ready with?

<u>EXTENSION</u>:

• What if the owner of each decided to give you a volume discount? At the supermarket, they give you a 10% discount. At the deli, they give you a 20% discount. Now who has the better deal? Explain and show your thinking!

• What if you wanted exactly thirty Gatorades? Would you still buy them from the supermarket? Explain and show your thinking!

<u>MODIFICATION</u>:

• Additional question -- how much would one Gatorade cost at each store?

• For those who really struggle -- change the amounts so that the numbers are easier to work with. For example:

-Supermarket: 10 for $15

- Deli: 5 for $8

LESSON PLAN: THE DRINKS PROBLEM (VI)

What are the two or three big ideas for the sharing time?

The one big idea is: how can we make a fair comparison? How can we compare the prices at these two stores in a way that shows that one store is cheaper unambiguously? This is where the different strategies in comparison come in. Some children will have compared the prices of a single Gatorade, while others will have chosen different "base" amounts. This discussion can then lead to the traditional algorithm of creating fractions with common denominators to compare them. But here, students will understand the WHY behind the algorithm -- the relational understanding for which we strive.

The other big idea is that fractions are connected to division. This will come into play in the next problem -- the sandwich problem -- in this way, creating an educative experience that lives on in future experiences.

How will students thinking be made public?

Students will work in their small groups to create a poster of their solution method. The guidance given to them will be this -- if a stranger walked in and read our problem, and then read your solution, they should be exactly clear on how you solved the problem.

MATERIALS AND RESOURCES - ENTIRE UNIT

MANIPULATIVES

- Fraction towers
- Fraction strips
- Fraction circles
- Two-sided colored counters

FRACTION MODEL MATERIALS

- Percent grids (10 x 10)
- Decimal grids (tenths, hundredths, thousandths)

GENERAL SUPPLIES

- Scissors
- Tape
- Overhead projector
- Calculators
- Index cards
- Chart paper
- Colored pencils and crayons

RESOURCES - SAMPLES ONLY - INCOMPLETE LIST

See attached sheets:

- Grid patterns
- 10x10 grids
- Clock face fractions
- Decimal grids

MATERIALS AND RESOURCES - ENTIRE UNIT

MANIPULATIVES

- Fraction towers
- Fraction strips
- Fraction circles
- Two-sided colored counters

FRACTION MODEL MATERIALS

- Percent grids (10 x 10)
- Decimal grids (tenths, hundredths, thousandths)

GENERAL SUPPLIES

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- Tape
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CHAPTER 5:

CONCLUSION

CONCLUSION

This curriculum is not rocket science. It is not a revolutionary new way to explore fractions, decimals, and percents. In fact, much of it is built upon existing curricula. However, it does feature several key factors that allow for greater creation of relational as well as instrumental understanding. First, every experience is not an end in itself -- every experience is discussed, reflected upon, and relived in future experiences, with greater understanding coming every step of the way. Second, the curriculum balances both relational understanding *and* instrumental understanding -- its goal is that by unit's end, each student will be able to effectively and efficiently solve problems (instrumental), and also be able to explain why their reasoning works (relational). Third, kids talk about math. A lot. A large portion of *every* lesson is the math congress, where ideas are discussed, debated, defended, critiqued. Fourth, the mathematics is grounded in real-world problem-solving contexts. The final exploration brings together all of the skills, knowledge, and problem-solving abilities and challenges students to apply them in solving AND explaining their solutions to complex problems.

It is my hope that I have taken the thinking and work of many others and made it my own. In this way, I hope to help uncover the complex and wonderful world of fractions, decimals, and percents with my students.

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