Spring 2017

What's in a Game? Exploring the Depth of Concept Teaching Games in the Math Classroom and the Use of Games as a Tool for Beginning Constructivist Teachers

Leah Silver

What’s in a Game?

Exploring the Depth of Concept Teaching Games in the Math Classroom and the Use of Games as a Tool for Beginning Constructivist Teachers

Leah Silver

Bank Street College of Education

Spring 2017

Advisor: Dr. Hal Melnick, PhD

Abstract

What’s in a Game?

Exploring the Depth of Concept Teaching Games in the Math Classroom and the Use of Games as a Tool for Beginning Constructivist Teachers

Leah Silver

This project explores the definition and depth of Concept Teaching Games in the elementary classroom. A series of four experiences, crafted for new teachers, lays out different elements of Concept Teaching Games and provides justification for their use. The first experience includes learning the definition of Concept Teaching Games, exploring their use as a tool for teaching new concepts to teachers. The second experience is a closer look at specific games and how they relate to the concrete to abstract spectrum. Learning where games fall on this spectrum helps beginning teachers evaluate the use of games in their classroom. The third experience provides a framework to use games as a form of formative assessment in the classroom. And the final experience demonstrates the necessity of varying our practices for the specific group of children in each classroom, with specific regard to the importance of making mathematical discourse accessible to students with language variations.

Table of Contents

Introduction

It was the fall of 2016, midway through my second semester as Hal Melnick's teaching assistant (heretofore referred to as TA) in the Math for Teachers in Diverse and Inclusive Settings course. I arrived to Hal's office ready to discuss and prepare for our afternoon class. "Leah, change of plans. I just have to do a game night," Hal declared. It's the place value night, where we explore the progression of place value through the grades, Hal explained. "What better way to explore these concepts than with some great games?"

In my experience as Hal's TA in three different semesters, I have seen many teachers re-engage in math through experience and inquiry. Many teachers enter the course afraid of the prospect of being in a graduate school math course. Many teachers have written about their negative math experience as children. Hal taught me that teachers teach the way they feel. Do we want teachers teaching their fear of math, or their love of math? How can we help teachers teach to provoke curiosity and bring out the joy in learning math? Games, I believe, are one answer.

Games, at their core, are fun. Games, made up of a constructed set of rules and regulations with a challenge to follow, invite people to figure them out, to strategize, to team together. In my experience in assisting the Math for Teachers course, I have seen games as one of the most accessible ways into constructivist learning and teaching (explained further in the rationale). Marilyn

Burns, a prominent math thinker, describes the use of games in the classroom, "Games are ideal for engaging students independently and productively when they have extra time. And games are effective options for the paper-and-pencil practice that worksheets often provide" (Burns, 2007, pg 89).

Throughout my experience as Hal's TA, I developed a way of presenting concept teaching games to new teachers, answering the following questions:

- What is a concept teaching game, and what makes a good game?
- How can we vary concept teaching games to the different abilities, variations, and levels of the children we work with?
- How can teachers use games as an assessment tool?

In the pages that follow, I will lay out my presentations for each of these topics. For the purpose of this project, these sessions were designed not only to be used within the Math for Teachers course, but can be used as professional development sessions for teachers in different settings. My hope is that new teachers who are hoping to implement constructivist practices into their classroom can look at these resources, understand the depth of games, and feel comfortable beginning to use games in their classrooms.

Rationale

A simple search for “math games” on Google will yield over twenty two million results! Clearly, the market is saturated with resources for parents and teachers. That sounds great, right? More games, more play, more math fun for children. But, are all games really created equal? While it’s heartening to see so many games out there, a good game (and the children at play) deserves careful consideration. This Integrative Masters Project serves to outline the elements in a quality concept-teaching game, and help teachers understand the depth of a good concept teaching game.

In this rationale I discuss four key elements:

- *Why experience and play are important in the math classroom*
- *How games support and enhance a constructivist learning environment*
- *Why games serve as a meaningful entree into constructivist teaching for beginning teachers*

Why Experience and Play are Important in the Math Classroom

Experience is a powerful and important piece of a child’s life. Experience is inevitable, but it is up to teachers to craft and shape experiences that promote a desire to further learning and asking questions. John Dewey, in his book *Experience and Education (1938),* has helped teachers think about the kinds of experiences our children deserve. In distinguishing educative experiences from miseducative experiences, he writes, “...if an experience arouses curiosity, strengthens initiative, and sets up desires and purposes that are sufficiently intense to carry a person over dead places in the future, continuity works in a very different way. Every experience is a moving force. Its value can be judged only on the ground of what it moves toward and into” (Dewey, 1938, p. 38). Here, Dewey helps us establish the criteria for a positive experience. Experiences need to arouse curiosity, strengthen a student’s initiative, and build a desire for more educative experiences. Concept teaching games have the potential to fulfil all of these categories. Engaging concept teaching games invite students into play, encourage students to strategize and, hopefully, leave students wanting more.

Dewey helps us think further about the planning required for implementing these kinds of experiences. He writes, “It is not enough that certain materials and methods have proved effective with other individuals at other times. There must be a reason for thinking that they will function in generating an experience that has educative quality with particular individuals at a particular time” (Dewey, 1938, p. 46). In a constructivist environment, a teacher must understand the developmental stages of the age group, and the specific needs and desires of students. In a constructivist classroom, games are not just printed out and given to children. Teachers select specific games based on each student’s desires and needs. This not only requires a knowledge of the students, but a deep knowledge of the game. “The educator is responsible...for a knowledge of subject-matter that will enable activities to be selected which lend themselves to social organization, an organization in which all individuals have an opportunity to contribute something, and in which the activities in which all participate are the chief carrier of control (Dewey, 1938, p. 56) In order for experiences to drive further curiosity, the teacher must deeply understand the task at hand. Before game play, teachers must play the games themselves. What questions emerge? What strategies might students use? Are the materials suitable for the game? Are additional tools required? Games (and the children engaged in playing games) are experiences that deserve time and thought before introducing them to children.

How games support and enhance a constructivist learning environment

Games have the potential to encompass so many elements of a constructivist classroom. James Hiebert outlines five dimensions of a constructivist classroom. Let's examine each, and how games would fulfill or enhance these dimensions. The first, is the *nature of classroom tasks.* According to Hiebert, tasks should connect with students at their level, leave students thinking, and make math problematic. Doyle (1983) writes, as cited in Hiebert (1997) , "Students learn from the kind of work they do during class, and the tasks they are asked to complete determines the kind of work they do" (pg. 17). The nature of the task or activity is crucial to how students will develop their perceptions of math. If students are only asked to do paper and pencil tasks, teacher led and student solved, then students will internalize that mathematics is about these kinds of tasks. Games, therefore, are essential to helping our students develop positive perceptions of what mathematics is. During a concept teaching game, students are happily engaged in mathematics that is probing their curiosity, encouraging engagement with peers, and asking for further play.

The second dimension is the *role of the teacher.* Teachers play a crucial role in creating a classroom culture of collaboration, reflection and communication (Hiebert et al p 39). Teachers also play a crucial role in selecting tasks that are meaningful and promote flexible thinking. A concept teaching game only enhances this dimension. During a concept teaching game, the teacher is removed from being the only source of knowledge; students are engaged with each other and the materials. The tools become the teachers. Further, concept teaching games require communication amongst peers. Materials are shared, strategies are discussed; a game cannot be played individually.

The *social culture of the classroom* is the third dimension of a constructivist classroom. Hiebert explains, “When students are working on their own, they can get locked into thinking of a problem in one way, and developing only one method for solving it” (p. 45). When working together, on the other hand, students have access to more methods, students are more likely to verbalize their mathematical thinking, and more difficult problems become more accessible. During game play, students who may not typically feel comfortable participating in a whole class will have the opportunity to verbalize their thinking in a smaller, safer environment. As mentioned earlier, students are required to work together during game play, elevating their flexible thinking and collaborative problem solving skills.

The fourth dimension is *mathematical tools as learning supports.* Students deserve multiple opportunities throughout the week to engage with mathematical tools. In order for students to construct meaning for and with tools, students need multiple access points with the tools. While a game should not be the first time students engage with the tools used during the game (dice, pattern blocks, ten frame cards, etc…), a game provides a structured opportunity for students to constantly engage with a tool, over and over, within a small time frame.

And the final dimension is *equity and accessibility.* Every student deserves to learn math with understanding (Hiebert, 1997). Every student, regardless of gender, ability, and learning variations deserves to feel engaged and challenged in mathematics. Games relate here in two ways. Firstly, games can serve as a natural equalizer; a hearty concept teaching game can engage learners on multiple levels. Some may have more access to the more nuanced strategies while others may just be learning a concept. A great concept teaching game can be played over and over, with new meanings developed in each round. Games are also a great way for teachers to target specific skills for specific students. Rather than having students complete more worksheets to understand a concept, a great game can serve as another teacher in the room (Hal Melnick, personal communication, Spring 2017).

Rationale for Using Games to teach Beginning Teachers about Constructivist Education

In my experience as Hal Melnick's teaching assistant for three semesters, I have seen many students re-engage with mathematics and learn to love and appreciate learning and teaching math. Many students in this class are in settings where they hope to be able to bring some of their learning from this class. Some settings are more constructivist-based than others, and some are quite traditional. Additionally, most of the teachers in this course are assistant teachers and student teachers, not fully in control of what or how information is taught. Good games, which encompass all five dimensions Hiebert outlines, are an opportunity for beginning teachers to play the role of facilitator, and to bring these dimensions into their settings. Games can be introduced and played in one day, and many beginning teachers who not have control over every day in their curriculum can still teach a game in one lesson.

Clements and Battista open their article on Constructivist learning with the following quote, "In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding” (MSEB and National Research Council 1989, 58). The Math for Teachers course at Bank Street embodies this quote. Rather than explaining how to teach through a series of lectures and papers, students walk into this course each week greeted by materials, perplexing problems, and interesting games. This course models, through experience, what learning math can feel like. And this, in turn, helps teachers craft constructivist experiences for their own students.

Further, in game play, the role of the teacher is no longer as teller or as the source of information. Games require teachers to have and practice restraint: restraint from moving pieces, giving answers, and asking leading questions.

What You’ll See in the Pages that Follow

This project is compiled of four experiences I’ve compiled to engage beginning teachers in the exploration of the use of games in the classroom. These experiences are best when spread out over some period of time, rather than all in one day. This way, teachers can go back to their classrooms and explore ideas in between sessions.

The four experiences are as follows:

- *What’s in a Game? Defining Game Play in Our Classrooms*
- *Digging Deeper into Concept Teaching Games (an exploration of the concrete to abstract spectrum)*

- *Using Games as Formative Assessment*
- *Using Number Talks to Engage Students with Language Variations in Mathematical Discourse*

The final component of these experiences is for teachers to create their own concept teaching games. In each semester of Hal's Math for Teachers course, new teachers have gone on the journey of creating their own game. To be clear, this does not mean that teachers should be creating all of their own games in the classroom; the purpose of this exercise is for teachers to fully understand the components of a great game and take this new appreciation toward other pre-made games they may use in their classrooms.

Description of Each Experience

There are some common threads and ideas between each of these sessions. Firstly, in each of these four experiences participants will play games themselves, to help them justify the use of games in their own classrooms. It is important for participants to have these experiences themselves, to develop a strategy, to experience a hopeful moment, and to experience the pleasure in game play. Participants will also stop to reflect on the experience of game play, and dissect the components of the game to further understand its potential use in the classroom. Below is a description of each session: a general overview as well as some reflection based on my experience using these materials with students *in Math for Teachers. Rather than show each individual slide in the description of each session, a selection of the slides is included to give a reader a feeling for each presentation. The complete slide sets are included the appendix.*

The First Experience

What's in a Game? Defining Game Play in Our Classrooms

In order for teachers to understand the multiple uses and the depth of game play in their classrooms, I believe they should first discuss the elements of a game. Before this session, it would be helpful if teachers read Melnick's article, *The Concept Teaching Game, a Rationale* and Clements and Battista article on constructivist teaching and learning:

Recommended Reading Before this Session

Melnick, Hal PhD, (2000, 1987) *The Concept Teaching Game: A Rationale* In Thought and Practice; the journal of the Graduate School of Education . Volume 1, Number 1, Spr 1987.

Clements, D. H., & Battista, M. T. (2002). Constructivist learning and teaching. In National Council of Teachers of Mathematics, *Putting research into practice in the elementary grades: Readings from journals of the NCTM* (pp. 6-8). Reston, VA: NCTM.

First, participants will play *Action Fractions*, a game developed by the fourth grade students of Hal Melnick while working in an inner city public school in Queens, NY. This game unearths many concepts for participants to construct, such as the meaning of a fraction, equivalent fractions, improper fractions, and operations of fractions. Further, the concepts within this game are often not made accessible to students in a school that emphasizes procedural thinking as opposed to concept-oriented thinking. The goal in a constructivist or concept oriented classroom is to help children understand the connections needed in developing computational fluency or mathematical proficiency. For example, in procedure-centered math curriculum, students beginning their fractions unit first explore “unit fractions,” fractions that are one part out of a certain amount of equal parts. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, etc… The numerator is 1 in a unit fraction. In Action Fractions, students are exposed to many different fractions, including fractions that are often called “improper”. These fractions have a larger numerator than denominator, making the fraction larger than one whole. These fractions are often taught only after students have learned other fractions. This game demystifies the idea, for teachers, that fractions must be taught in this specific order. Lastly, students in a procedure oriented classroom often learn equivalence only after adding fractions of the same unit size (ex: $\frac{1}{6} + \frac{5}{6} = 1$). By playing Action Fractions, participants are exposed, from the onset, that there are so many different ways to make a whole. In this game that bases its teaching in concrete tool use, the traditional trajectory of first adding or subtracting fractions with like denominators is replaced with adding physical amounts, which may or may not have like denominators. As we move along teaching skills for writing operations without materials, the need emerges to establish rules or procedures for changing denominators later. At this introductory juncture, the ‘procedures only’ is avoided, and not needed. That will come in time.

This game, along with all games, must be experienced by the facilitator before using it with a group of students. Without experiencing it first, the facilitator will have little sense of the questions a student might pose or the meanings a student might discover. Further, the facilitator of these games need to understand what students should know before playing the game, and what kinds of experiences should follow. Playing the game first helps give the facilitator a sense of where and how this game fits into the child's mathematical learning.

For participants concerned with Common Core alignment (and everyone else!) we can see that Action Fraction can actually fulfil almost every fourth grade fractions standard. It may be useful to look at the whole list and ask participants which of the standards Action Fraction addresses. Here are some Common Core State Standards that are addressed in playing Action Fractions:

Extend understanding of fraction equivalence and ordering.

CCSS.MATH.CONTENT.4.NF.A.1

Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

CCSS.MATH.CONTENT.4.NF.A.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions.
CCSS.MATH.CONTENT.4.NF.B.3

Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

CCSS.MATH.CONTENT.4.NF.B.3.A

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.MATH.CONTENT.4.NF.B.3.B

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*.

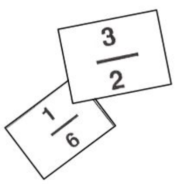
CCSS.MATH.CONTENT.4.NF.B.3.C

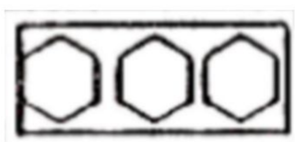
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

http://www.corestandards.org/Math/Content/4/NF

Play Action Fraction at your table

Each player needs a game board. Each table needs a set of fraction cards





Your goal is to fill each of the three hexagons. Each hexagon is considered “one whole”.

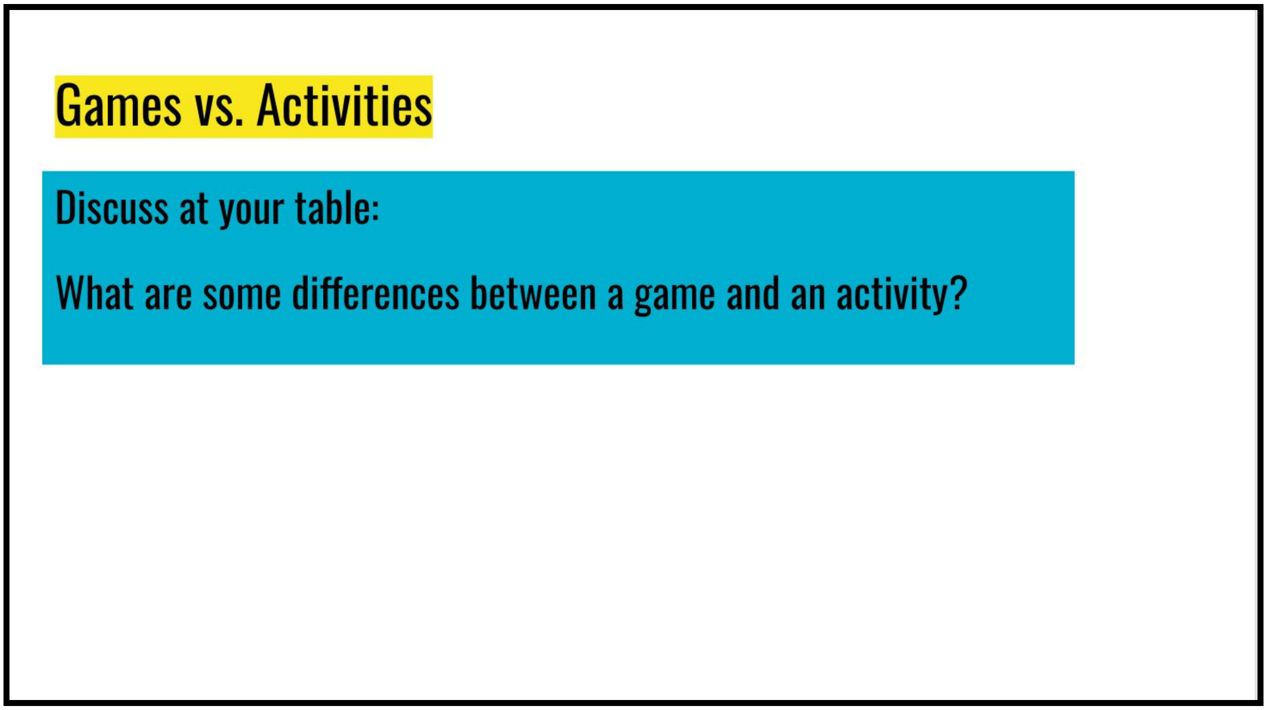
Take turns picking cards and fill the corresponding amount of space on your game board. The first person to fill their game board wins.

Once participants have played this game, they will discuss some ideas they have about the difference between a game and an activity. Understanding this difference is crucial. In previous sessions, participants have described a game as more fun than an activity, and more competitive than an activity. Activities may have no end in sight, where as games are more fixed experiences.

Often in Math for Teachers, we hear students describe a “game” their cooperating teacher played, only to discuss it further and realize it is really an activity. Once participants have had the opportunity to discuss some ideas they have about the difference between games and activities, they will look at the NCTM Criteria of a Game (*Learning and Mathematics Games, NCTM Monograph #1, 1985)*:

1. A game is freely engaged in.
2. A game is a challenge against a task or an opponent.
3. A game is governed by a definite set of rules. The rules describe all of the procedures for playing the game, including goals sought; in particular the rules are structured so that once a player’s turn comes to an end, that player is not permitted to retract or to exchange for another move the move made during that turn.
4. Psychologically, the game is an arbitrary situation clearly delimited in time and space from real life activity.
5. Socially, the events of the game situation are considered in and of themselves to be of minimal importance.
6. A game has a finite state- space (Nilsson, 1971). The exact states reached during play of the game are not known prior to beginning of play.
7. A game ends after a finite number of moves within the state-space.

They will compare these ideas to what they discussed. Further, it’s important to distinguish between a math goal and an activity. An activity may accomplish a goal, but an activity is not the goal or purpose itself. In order to tease out these semantics, it’s important to play these games so that participants can articulate the concepts embedded in the games they’ve experienced.



Hopeful Moments

In my second semester assisting the Math for Teachers Course, as students shared their Concept Teaching Games, I wanted to figure out the commonalities of the particularly fun and exciting games. As I played these games, I noticed participants saying, “I hope I get a….” or “I hope I…”. To me, the “hopeful moment,” as I began to call it, was more exciting (and mathematically enriching) if it related back to the concept. For example, in Action Fraction(s), here are some examples of *hopeful moments* participants might experience:

- I hope I get a larger fraction because I need to fill a lot of space

- I hope I get a small fraction because I only have a little space left
- I hope I get ⅙ because that's what I need to complete this hexagon

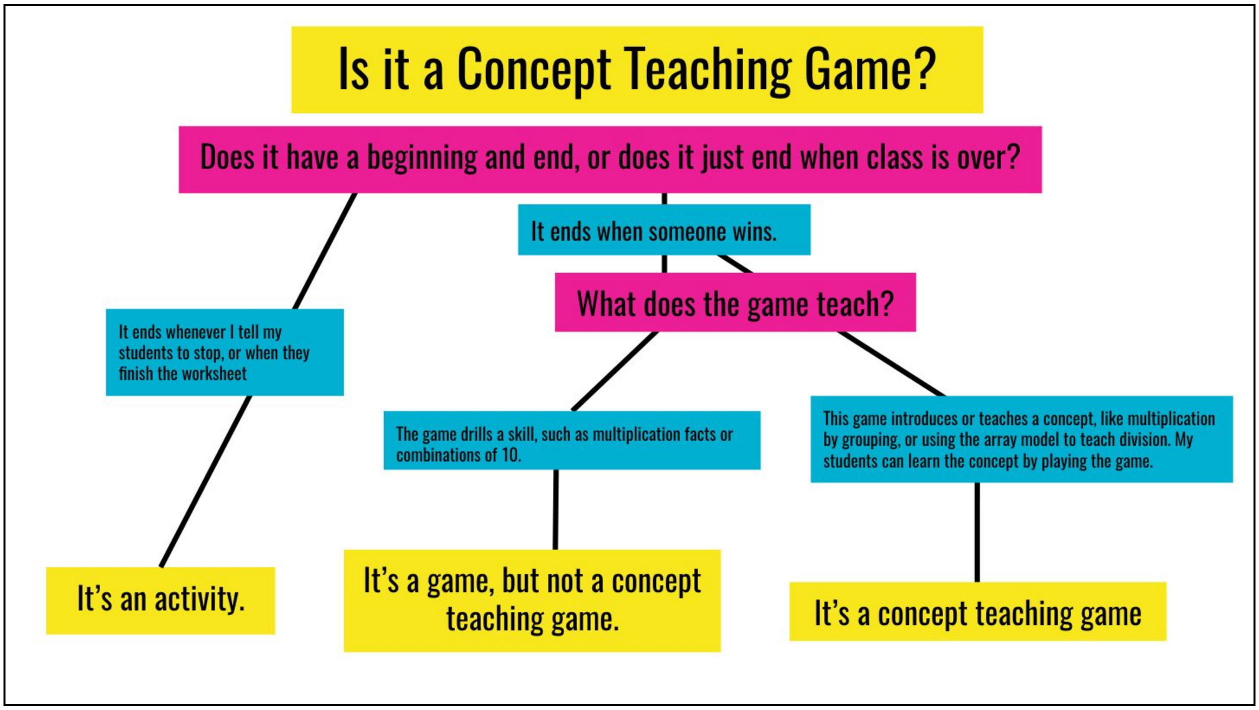
These *hopeful moments* relate directly back to the concept the game intends to teach. Some games may have *hopeful moments* that don't directly relate to the concept. These games generally aren't as rich in mathematical concepts. An example of this kind of game may be a board game, where students roll dice to propel them the corresponding amount of spaces. Depending on where the student lands, he or she may have to draw a card to determine the type of addition task they have to complete in order to stay on their spot. The goal of the game is to go all the way around the board. Thus, the addition concepts in the game are totally removed from the structure of the game. And the structure of the game (a board game) is merely a mechanism for students completing addition tasks. A student may hope to get a high number on the die, but this hopeful moment has nothing to do with the addition concepts. When the *hopeful moments* relate back to the concept, it provides even more access points into the concept. Further, depending on where students are in their understanding of the concepts, their hopeful moments will sound different. In Action Fractions, some students will be hoping for "small" or "big" fractions; others will hope for a shape they can break up into smaller fractions, others will hope for a certain amount more than their opponent. Understanding these hopeful moments and the range of hopeful moments helps teachers provide rich and meaningful games for their students.

Criteria of a Game

In the NCTM monograph, an instructional game of this sort is defined as having seven criteria.

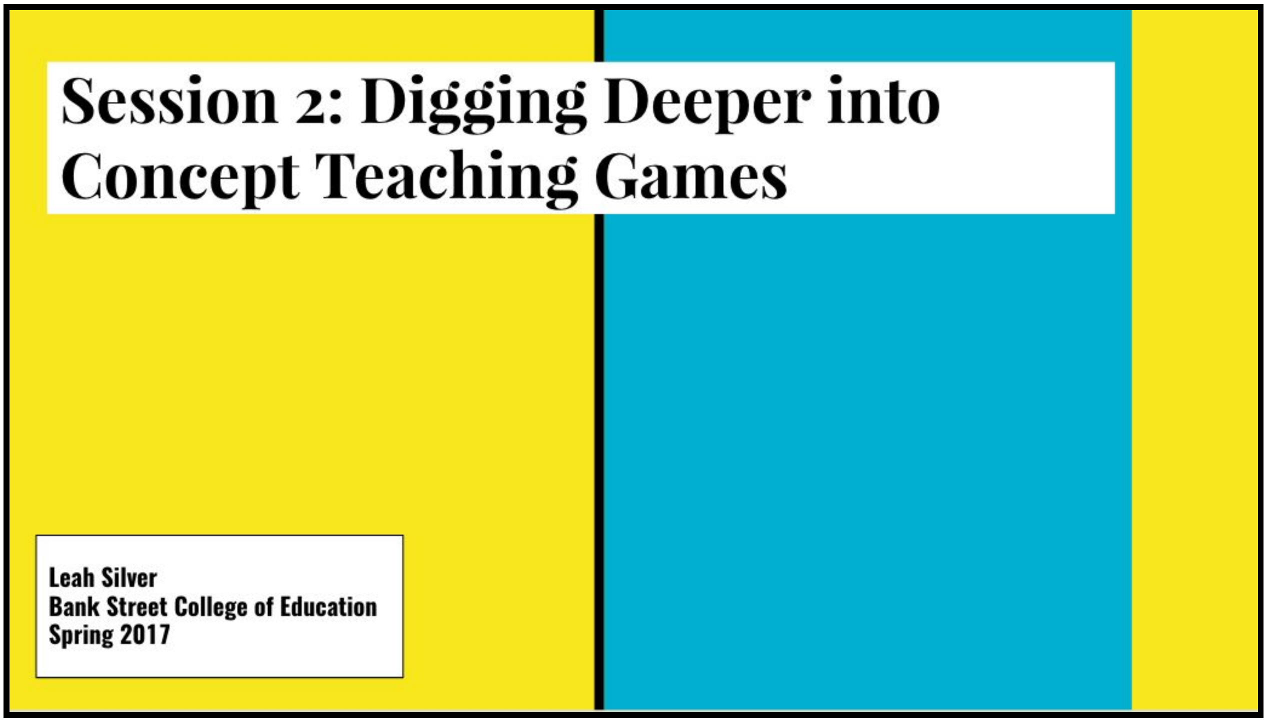
1. A game is freely engaged in.
2. A game is a challenge against a task or an opponent.
3. A game is governed by a definite set of rules. The rules describe all of the procedures for playing the game, including goals sought; in particular the rules are structured so that once a player's turn comes to an end, that player is not permitted to retract or to exchange for another move the move made during that turn.
4. Psychologically, the game is an arbitrary situation clearly delimited in time and space from real life activity.
5. Socially, the events of the game situation are considered in and of themselves to be of minimal importance.
6. A game has a finite state- space (Nilsson, 1971). The exact states reached during play of the game are not known prior to beginning of play.
7. A game ends after a finite number of moves within the state-space.

Participants will end the session by reviewing the elements of a concept teaching game. It would also be helpful if participants have the opportunity to play a contrasting game that is clearly not concept teaching but just a skill reinforcement game. While playing a concept teaching game, the concept has to be re-experienced over and over again. This is not true of skill reinforcement games. Participants will discuss, in partnerships, potential ideas and concepts for a Concept Teaching Game in their own classrooms. By the end of this workshop, students will develop their own concept teaching game.


Is it a Concept Teaching Game?
Does it have a beginning and end, or does it just end when class is over?
It ends when someone wins.
It ends whenever I tell my students to stop, or when they finish the worksheet
What does the game teach?
The game drills a skill, such as multiplication facts or combinations of 10.
This game introduces or teaches a concept, like multiplication by grouping, or using the array model to teach division. My students can learn the concept by playing the game.
It's an activity.
It's a game, but not a concept teaching game.
It's a concept teaching game

The Second Experience

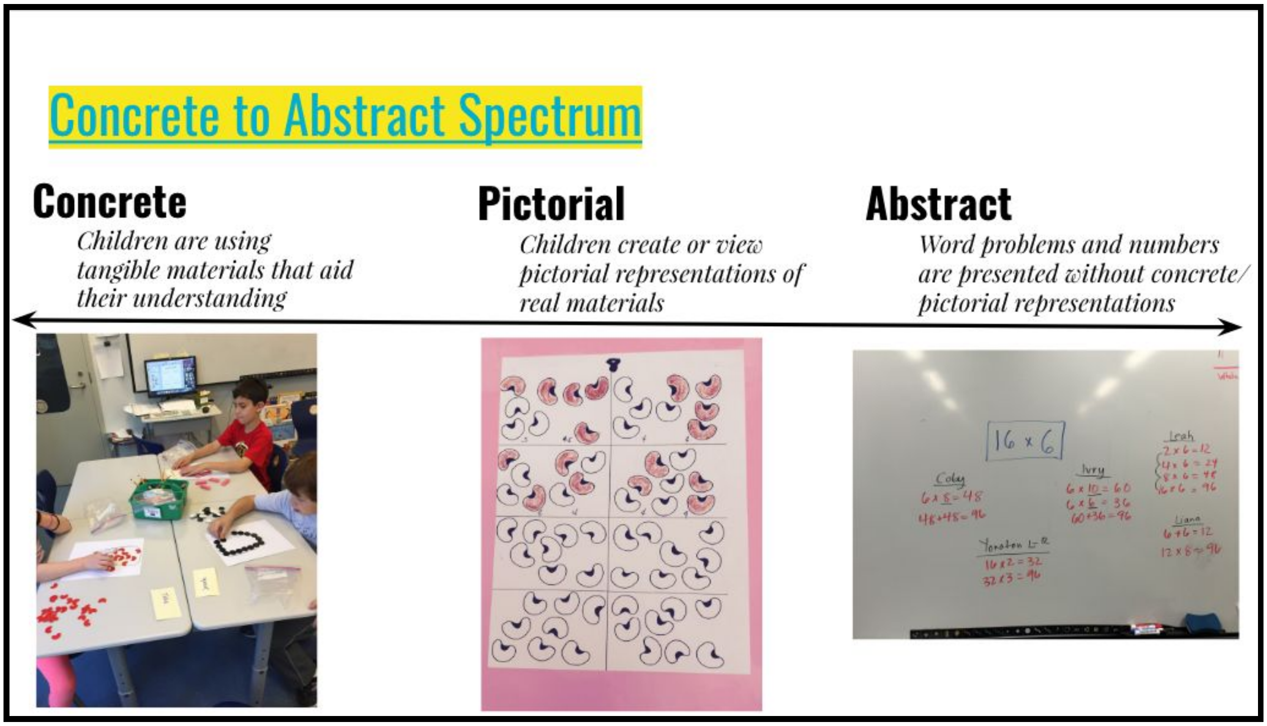
Digging Deeper into Concept Teaching Games



The goals of the second experience are to expand on the definition of games, and be able to classify games by what they help children to understand. After this session, participants will be able to look at a game and determine what their students need to know about the concept prior to playing the game.

This experience provides one framework for thinking about where this game falls on the developmental spectrum. It's important that participants know that this is just one way of classifying games, and not to get locked into this structure for every game or activity. Jean Piaget helped us think about how children acquire concepts. His work, breaking down children's developmental stages to concrete, semi-concrete, and abstract, have helped many math educators think further about their math teaching. One common framework is the concrete-pictorial-abstract framework, grounded in Piaget's theory of development. Here, we think about presenting concepts to students in the most concrete level-- with real materials. Next, in the pictorial stage, students create and view pictures relating to the concept. Finally, in the abstract level, students are ready to view standard number problems without the aid of visuals or materials. This should emerge organically and therefore come after experiences with real objects and pictures.

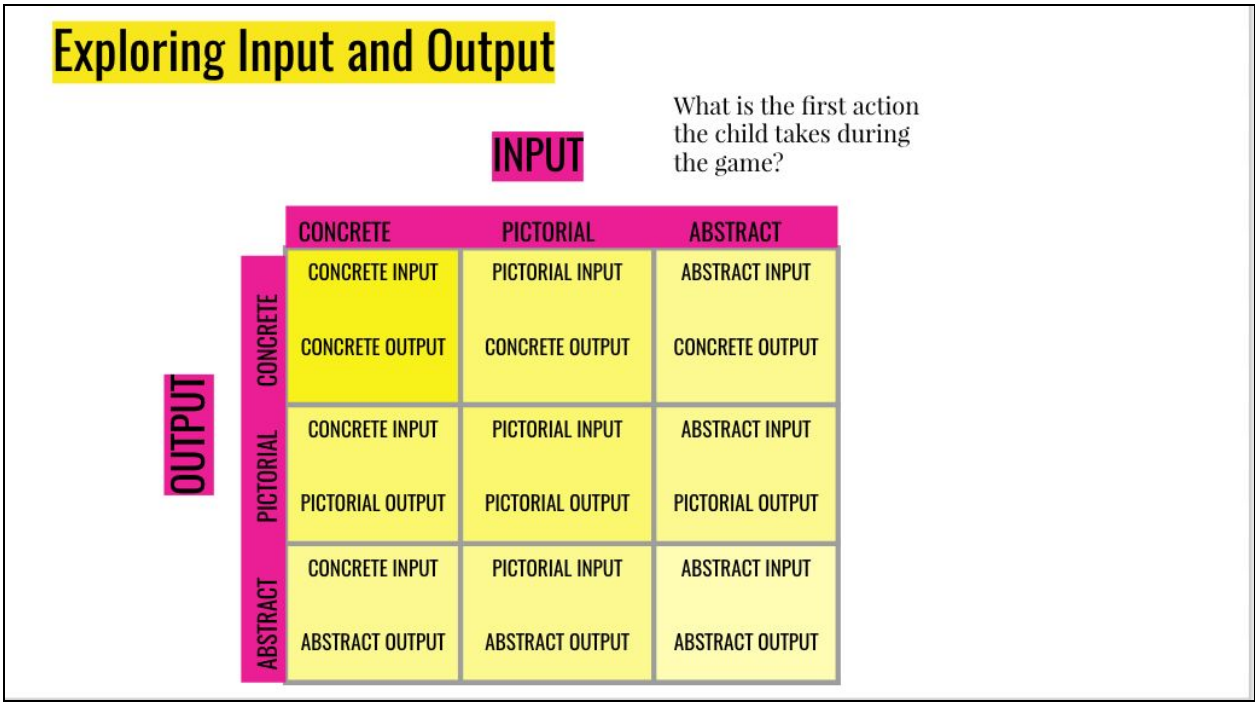
This framework can help us think about games, too. These concept teaching games are meant to do just that-- teach a concept. So, if the intention is to teach a concept, the game should at least partially operate in the concrete. This means that during the course of the game, students should have the opportunity to use real materials.



This experience further outlines this framework. An input/output graphic schema serves as one way to help adults classify games that fit within a concrete to abstract continuum. The input is what students have to do first. In the context of this session, we look at The Joining Board Game, a game that involves combining collections of unifix cubes into towers for numbers one to twelve.   The concepts focus in on joining two sets to combine to make sums and to offer players the right to then decompose or separate those two sets into smaller sub sets as needed to win. In Joining Board, the first move students make is to roll the dice. Is that concrete, pictorial, or abstract? When I first presented this question in the Math for Teachers course, it sparked a healthy debate. Some felt that dice were concrete; students actually point their pencil to each dot and feel the pips of the dice to count the numbers. Others felt that dice were actually pictorial. Similar to a tens frame, students learn to view the particular arrangements of dots on the dice as certain numbers. Viewing numbers greater than four requires students to subitize. Subitizing is the "process of instantaneous recognition of number patterns without counting" (Labinowicz, 1984, p. 105). Further, "Subitizers are able to recognize the number of objects in a group without counting each object. In order to subitize, the student must be able to visualize the quantity in a group, connect it with a numerical value, and verify by means other than counting that the connection is accurate" (*Losq, 2005, p. 310*). Practice with materials like dice and ten frames allows students to have a different framework within which to visualize numbers greater than four.

I drew two conclusions from this lively conversation. First, everything we do in the classroom depends on the group of children we're working with. Kindergartners who are working on recognizing numbers to ten may very well be using dice as concrete tools. Fourth graders who have already had a lot of experience with dice play may recognize the number patterns as representations of numbers, without counting the specific dots. Second, this framework is not set in stone. Like all structured frameworks, this requires a teacher to thoughtfully consider the previous experiences of his or her students, their developmental stage, and what experiences may come next.

The second move-- the response to the *input*-- which we'll therefore call the *output*, that students make in Joining Board is to take unifix cubes and build towers that correspond to the amount they rolled on the dice. This is most certainly *concrete*. Students are creating a concrete representation of a number.



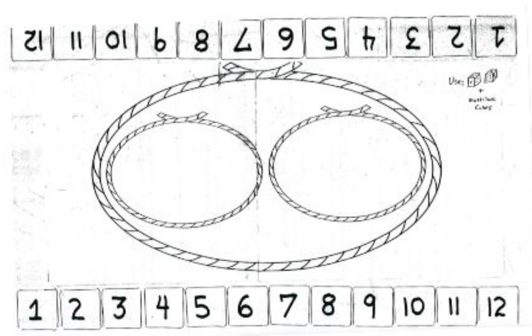
So, I would say that The Joining Board Game has a pictorial input and a concrete output, placing it on the outer band of the grid below. Games that are introducing or really teaching a concept should operate, somehow, in the concrete. Whether it's a concrete input or a concrete output, in order to learn a new concept students need the opportunity to engage with real materials. This chart helps us classify the games that we use as suitable for beginning concept learners, or at a different stage of their learning.

This isn't to say that games with an abstract input and output are bad games. In fact, in the following session participants will play a game called the 'Product Game' which is a very abstract game. Some of these games can be the most rewarding and fun to play. But before playing a game like the product game, we as teachers must consider what kinds of experiences will allow kids to make deep conceptual understandings.

Once participants have been exposed to this framework, they will play three different games and discuss what students need to know before playing each game. Then, they will determine where each game fits on the grid, and establish the concepts that each game can teach.

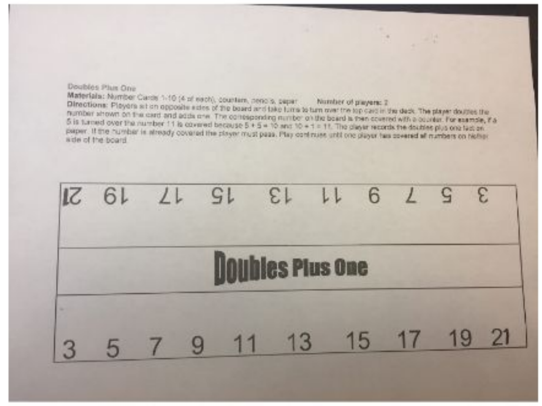
Joining Board

1. Where would you place Joining Board on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?



Doubles Plus One

1. Where would you place Doubles Plus One on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?


Doubles Plus One
Materials: Number Cards 1-10 (4 of each), counters, pencils, paper **Number of players:** 2
Directions: Players sit on opposite sides of the board and take turns to turn over the top card in the deck. The player doubles the number shown on the card and adds one. The corresponding number on the board is then covered with a counter. For example, if a 5 is turned over the number 11 is covered because 5 + 5 = 10 and 10 + 1 = 11. The player records the doubles plus one fact on paper. If the number is already covered the player must pass. Play continues until one player has covered all numbers on his/her side of the board.

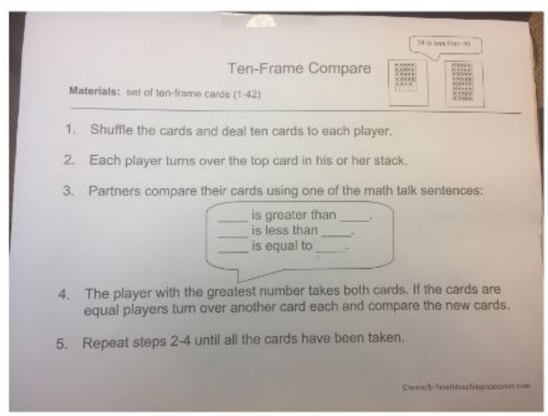
3 5 7 9 11 13 15 17 19 21

Doubles Plus One

3 5 7 9 11 13 15 17 19 21

Ten Frame Compare

1. Where would you place Doubles Plus One on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?


Ten-Frame Compare

Materials: set of ten-frame cards (1-42)

1. Shuffle the cards and deal ten cards to each player.
2. Each player turns over the top card in his or her stack.
3. Partners compare their cards using one of the math talk sentences:

____ is greater than ____.
____ is less than ____.
____ is equal to ____.

4. The player with the greatest number takes both cards. If the cards are equal players turn over another card each and compare the new cards.
5. Repeat steps 2-4 until all the cards have been taken.

Joining Board

We can learn about how to incorporate Joining Board into the classroom by looking further at its input and output. The first move is the roll of the dice, which as discussed earlier is arguably pictorial or concrete. The next move, or the output, is for students to gather the corresponding amount of unifix cubes and build towers accordingly. This step is most certainly concrete. Realizing that this game has a pictorial input and concrete output can lead us to figure out the best uses for this game. Students acquire new concepts through concrete experiences. A game like Joining Board with a concrete output, where students are using unifix cubes to build representations of numbers, is a great game to teach the concept of combining numbers and breaking numbers apart. A game that was more abstract, where students were only seeing written numbers and had to do the same task of putting the numbers together and breaking them apart, would only be possible for students to play *once* they've mastered that concept. Joining Board, however, helps *teach* this concept through the use of manipulative materials.

Doubles Plus One

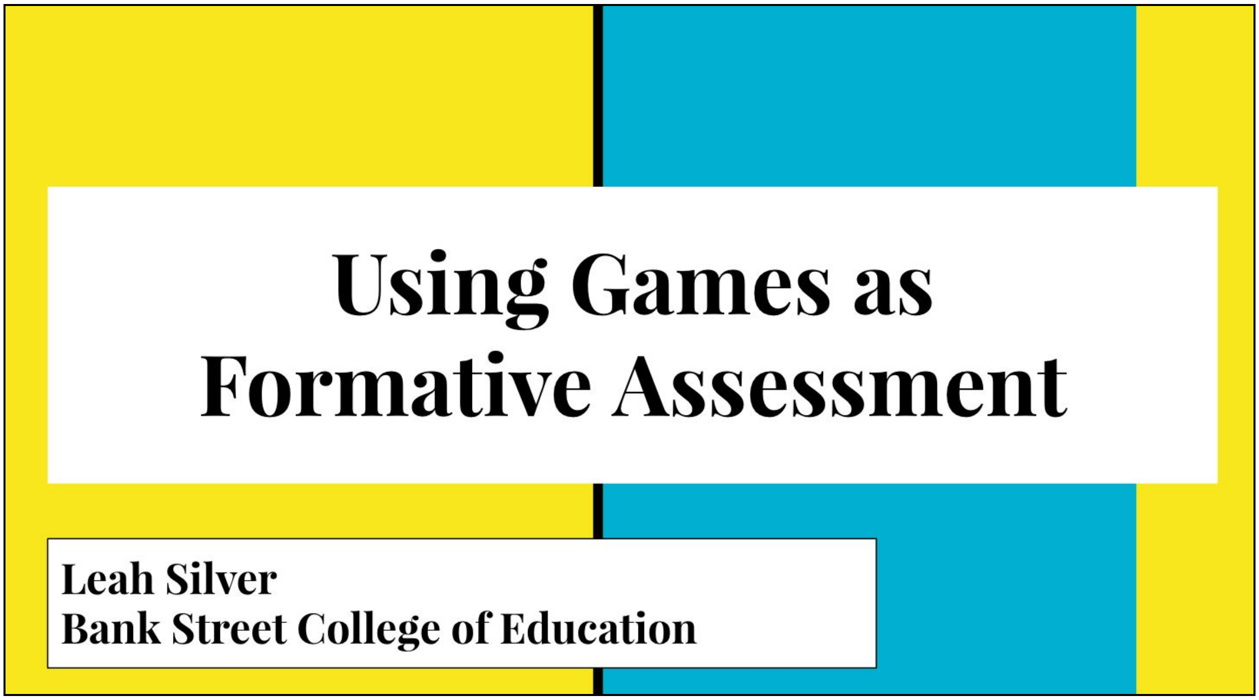
Doubles Plus One is a great game for reinforcing facts, but without additional supports it remains a pretty abstract game. Students roll the die, double the number, and then find that spot on the board. If students had to create that number with unifix cubes or some other tool, and then create an equal tower to double the number, and finally add one more, that would make the game slightly more concrete. This is a fun game for students to reinforce facts, though it isn't ideal for teaching a new concept. It's a game, but it's not a concept teaching game.

Ten Frame Compare

Playing this game through, we can see that it has a pictorial input and output. This game relies heavily on the use of a ten frame, a common tool teachers use to help students visualize numbers greater than four. While this is referred to as a tool, it should not be confused with concrete manipulative materials. It is a tool that helps students understand a concept, but pictorial tools like ten frames must not take the place of concrete materials. Once students have had concrete experiences exploring the value of single digit numbers and asking, "Is this greater or less than….?" and once students have had prior experiences using ten frames, they are ready to play Ten Frame Compare.

The Third Experience

Using Games as Assessment Tools

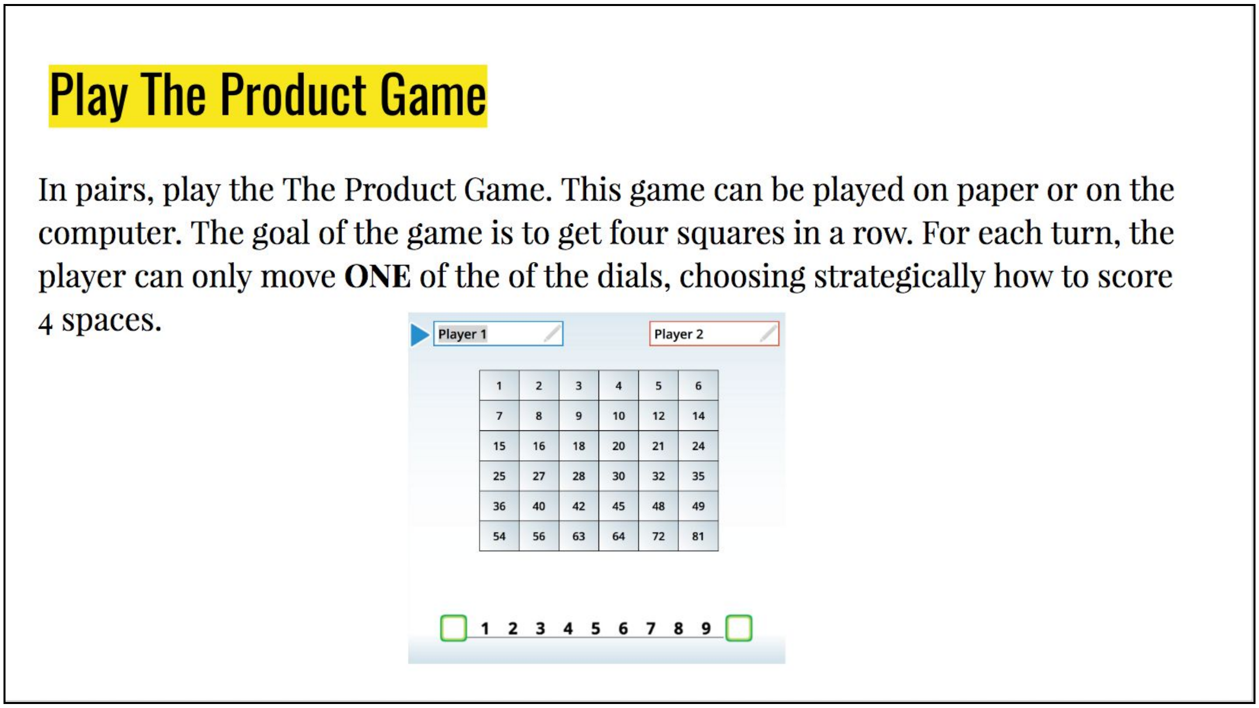


In the graduate course Math for Teachers in Diverse and Inclusive Classrooms (N-6) students often ask how to justify the kinds of experiences we advocate to their cooperating teachers, school administrators and parents. A question is often raised about assessment. It's important to broaden the definition of assessment from the traditional pencil and paper written tests, and to help teachers understand key differences between summative and formative assessments. Well crafted concept teaching games can serve to help a teacher see the areas where a student stumbles as well as areas where a student shows profound mastery. In this experience, participants will learn to assess during game play-- by observing and asking their students questions. The purpose of this session is twofold: first, to broaden participants' understanding of assessment, and more narrowly to understand one framework for asking questions during games.

Similar to the other frameworks used throughout these sessions, the framework for question asking in this session is one of many that can be used to think about assessment. It's far from the only framework, and may not be suitable to every game in every classroom for every teacher. I came across this framework in an article from *Teaching Children Mathematics,* the NCTM journal (Bahr & Bahr, 2017), and I really appreciated how the authors helped teachers break down types of questions and types of student answers.

Participants will begin this experience by playing 'Product Game,' an abstract game that helps students see the interconnectedness of multiplication and division, among other concepts. Participants will play the game, and then discuss the concepts embedded in the game. Teachers cannot adequately use a game (or any other tool) as an assessment unless they have experienced it for themselves and unless they can articulate for themselves what the purpose and educational value is in the experience. Furthermore, teachers cannot use games as a form of assessment the first time students have played the game. The

Product Game, I believe, is a prime example of this.



I choose this specific game because it is so rich with concepts and strategies. Each time I play, I notice that I develop another strategy or am thinking in a deeper way about the concept. I'm anticipating what my partner might do and that might impact where I want to place the next factor. I do believe there is a sweet spot to this game and have seen it be overplayed. Once both partners avoid placing the paper clip (if playing on paper) or slide the square (if playing on the computer) to a certain factor to prevent their partner from winning, the game becomes slightly less exciting (in my opinion). That being said, after playing a few times, teachers can begin to ask students questions indicated in Damon Bahr and Kim Bar's article, "Engaging All Students in Mathematical

Discourse" (2017) found within NCTM's journal *Teaching Children Mathematics,* about their choices to gain an understanding of what they know and don't yet know.

Once participants have played the game, they will discuss the concepts embedded in the game. Then, we'll look at one framework for asking questions. This framework breaks down questions into three C's: Comprehension, Connection, and Consensus.

Comprehension

Short Answer	Brief response, often an answer to a closed question	What did you get?
Brief statement	More information; still not elaborate	How did you get that?
Describe	Verbalization of thinking	How did you get that?
Elaborate or clarify	Adding more information to make things clearer	What did you mean by that?
Represent	Showing thinking in one or more ways	What would that look like in a picture or manipulative?
Compare	Determining whether two strategies or ideas are the same	Is ____'s thinking the same or different that ____'s thinking?

Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Each chart is broken down by the type of question, a categorization of student response, and an example question a teacher might ask. Participants will spend time discussing what kinds of comprehension questions they could ask students during *The Product Game.*

Connection

Relate	Determining how strategies, ideas or representations are similar or different	How is _____'s thinking the same or different than _____'s thinking?
Discern patterns	Recognizing and describing patterns across ideas, strategies and representation	What are you noticing as you think about all these…?
Discern structure	Recognizing and describing structures across ideas, strategies and representations	What are you noticing about the way these things fit together?
Reason	Explaining why the thinking is mathematically sensible	Why do you think your thinking about your idea is true (or strategy works or representation is accurate)?
Transfer	Using thinking in a new situation or context	Can you try this in this new situation?

Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Connection is the second tier of questions. These are great questions to ask as students become more familiar with the game. They have played it through a few times, and can begin to notice different patterns. A student playing Action Fractions may begin to notice that there are many different ways to fill a hexagon, a student playing the product game may realize that a few different combinations of factors will yield the same product. For example, to yield a product of 12, the paper clips can be place on factors 2 and 6, or 3 and 4. Having students verbalize the patterns they see or the reasons they make certain moves will help them toward the next step: consensus.

Consensus

Justify	Explaining why someone else's thinking is mathematically sensible or not	Why do you think someone else's idea is true (or strategy works or representation is accurate)?
Prove	Justifying truth, workability, or accuracy within a large domain	How do you know it is true in all cases?
Refine	Stating, solving, or representing more efficiently	Can you think of a more efficient way?
Generalize	Stating the net result of a proof precisely	How would you state what you have shown to be always true?

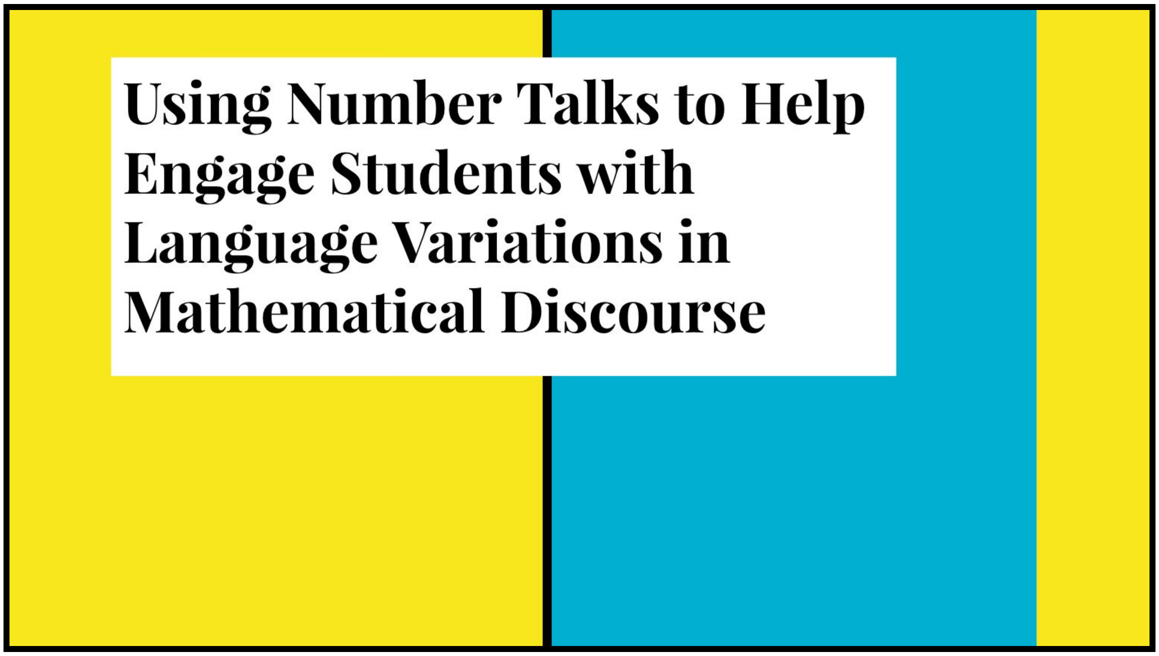
Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Consensus requires a level of understanding of basic concepts and an ability to fully communicate an understanding of those concepts. We can imagine that asking students to justify their thinking or the thinking of a peer during a game (for example,"Why do you think Mary decided to make that move?") wouldn't be a fair question unless the students had previous experience playing the game. When we first play a game, we're focused on ourselves, our moves, developing our strategy. Only once this happens can we start to look at the other's moves, their strategies, their game board, and build up our strategy even more. Here, we ask the participants, "What kinds of consensus questions can a teacher ask students who are playing the Product Game?"

Teachers may now have the impulse to see what their students know, to ask all of these questions. For these kinds of assessments, for both the student and teacher's sake, less is more. Asking the same group of students the same two questions will give the teacher a better understanding for what each student really knows. This way, teachers can better manage their own observations and recordings, as opposed to anecdotally recording whatever a student says, leaving much data for later interpretation.

The Fourth Experience

Using Modified Number Talks to Help Engage Students with Language Variations in Mathematical Discourse



The previous experience, about using games for questioning and assessment, requires students to have the receptive and expressive language skills to engage in high level mathematical discourse. While some students will be able to engage in this kind of conversation through practice and observing others engaging in conversation, other students will need much more language support to engage in high level discourse.

In my experience in the classroom, what is commonly referred to as Number Talks, transform a student's mathematical experience. Number Talks are conversations about one expression that a teacher poses on the board. Students have time to silently think about how to solve the problem, followed by a discussion where students share their strategies for the whole class. This allows students a safe and structured environment to verbalize their mathematical thinking. Timid students who don't believe in their own mathematical ability are often, in my experience, empowered by the flexibility and creativity Number Talks allow for. With three goals (accuracy, efficiency, and flexibility) and in a safely structured protocol, students share different strategies for solving the same problem.

COMPUTATIONAL FLUENCY

- Three key components: accuracy, efficiency, flexibility
- Understanding the meaning of each operation and how it relates to the others
  - *Ex: Addition is the inverse of subtraction*
- Knowing number facts and relationships
  - *Ex: 2 x 5 and how it relates to 2 x 50*
- A solid understanding of our place value system as a base 10 system
  - *Ex: A '5' means something different in the tens place than it does in the 100s place (50 vs. 500)*

This definition of computational fluency was adapted from the description from the Invesitgations in Number, Data and Space curriculum, retrieved from http://investigations.terc.edu/library/bookpapers/comp_fluency.cfm.

NUMBER TALKS

- Developed by Ruth Parker and Kathy Richardson
- Posing an abstract math problem such as 18 x 5 and asking students to solve the problem mentally.
- The teacher then collects the different methods and looks at why/how they work.
- For example a teacher may pose 18 x 5 and and that students solve the problem in these different ways.

20 x 5 = 100 2 x 5 = 10 100 - 10 = 90	10 x 5 = 50 8 x 5 = 40 50 + 40 = 90	18 x 5 = 9 x 10 9 x 10 = 90	18 x 2 = 36 2 x 36 = 72 18 + 72 = 90	9 x 5 = 45 45 x 2 = 90

This description of *Number Talks* was adapted from Jo Boaler's article, *Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts*

I believe that any teacher who wants to hear high level mathematical discourse from their students while they are playing games should also be doing Number Talks in their classroom. Experience with Number Talks give students the language and the structure for sharing their strategies with their peers.

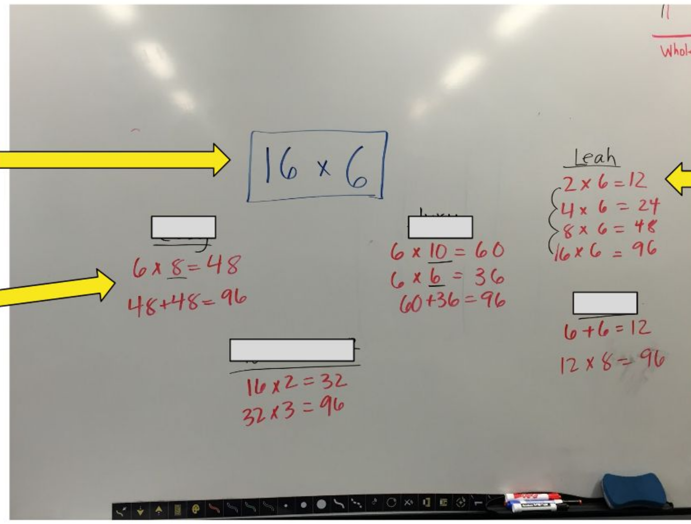
But for some, this protocol is still not enough. This presentation gives an outline (and an example) of what a Number Talk is, and outlines suggestions for modifying these Number Talks to give all of our students access to Number Talks.

A SAMPLE NUMBER TALK

This was a *Number Talk* I did with my students in the 2014-15 school year.

1. Teacher poses a problem and students think about it mentally

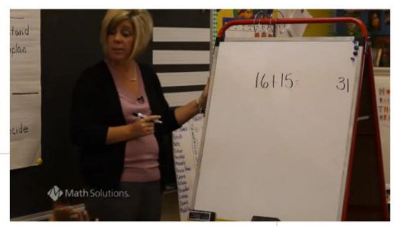
2. Students share various strategies



3. Sometimes the teacher shares a strategy, too!

Discuss: What language skills must a student have in order to participate in a Number Talk?

LET'S WATCH A NUMBER TALK



[*Click here to watch a 2nd grade Number Talk*]

As you watch, note the following key elements of a *Number Talk:*

- The structure (who talks when, how students share strategies, etc…)
- Teacher response to students
- How Number Talks promote computational fluency (efficiency, accuracy, flexibility

Discuss: In what ways does this structure support a child with language variations? What are the roadblocks for a student with language variations?

WHAT ARE THE POTENTIAL ROADBLOCKS FOR A STUDENT WITH LANGUAGE VARIATIONS?

RECOMMENDATIONS

EXPRESSIVE

Post sentence frames to help students share their strategies (see resources slide for a print-out)

Start with a "think aloud," modeling how a student might share their answer

Incorporate teaching of Visualization, or how to create a picture in one's mind

Call on a student to read the problem out loud first

Think, pair, share

BOTH

model a "think aloud"

vary group size, sometimes have small groups

Begin with problems that won't be computationally challenigng to students; the language is the new variable, not the math

RECEPTIVE

Have a student repeat the problem

Use manipulatives

Other visual cues

- concrete representation of the problem (using manipulatives, people)
- pictorial representation of the problem (draw a picture, use ten frames)

The Culmination

The Creation of Concept Teaching Games

Each semester in Math for Teachers, one of the assignments is to create a Concept Teaching Game. The requirement is not to have a fully finished game, but rather to have the experience of taking a concept and developing a game out of it. In my second semester assisting this course, I noticed some students presenting games while others' creations were really more activities. Hal and I then worked on creating more of this distinction during the semester, using the checklist below. This checklist also takes into account all of the experiences participants will have had. This checklist will help participants create interesting concept teaching games, taking into account not only the difference between a game and an activity, but also games with hopeful moments and more skill drilling games.

In an ideal setting, participants will have the opportunity to share their games with each other, and have the chance to ask questions and provide feedback to each other. The criteria below also serve as a helpful handout as participants are viewing each others' games.

Checklist for Teacher-Created Concept Teaching Games

___ My game has a beginning and a clear end (kids know when it's over)

___ My game introduces or teaches a *concept*, rather than drills a skill

___ My game encourages child to child interaction and engagement in mathematical discourse (the sharing of ideas)

___ My game is intellectually engaging for children

___ I can identify "hopeful moments" my students may experience during this game (and, ideally, these moments relate back to the core concept taught)

___ This game can be used as a formative assessment tool (you can learn what kids know and don't know about the concept)

Conclusion

I was not known to be a strong math student as a child. While I enjoyed some math activities in my constructivist school setting, my experiences varied from year to year. I remember being placed in accelerated math in ninth grade after a fun and successful year in Algebra, only to be demoted back to “regular” math after that challenging year in geometry.

And yet, here I am.

I was drawn into math once again in college, when I took an exciting and experiential course for future teachers. Entering Bank Street one year after my college graduation, I met my advisor, Hal Melnick, and my journey towards becoming a math teacher truly began. I felt that the math classroom had the most opportunities for me, a beginning teacher, to experiment with teaching practices and experiences. I could see the way students were thinking by the questions they were asking, the moves they were making in a game, and the strategies they were sharing. I realized then that what happens in the math classroom does not stay in the math classroom; as a math teacher I had the potential to help my students become more flexible thinkers, stronger problem solvers, and more confident participants in any kind of group setting. More and more over my time at Bank Street, as I’ve witnessed our nation’s leaders unable to communicate productively, respectfully, or successfully, I’ve seen that the constructivist math classroom can be the place for students to learn these crucial skills.

To me, promoting constructivist practices in the math classroom has become a form of social activism. Students who feel ashamed and embarrassed in the math classroom deserve to have doors open, deserve positive experiences, and deserve the chance to share without this fear. I've had students enter my third grade classroom already feeling turned off by math. "I'm not a math person," they might say. "I'm slow." "I'm stupid."

This just doesn't have to be. Through my experience working with these students, I came to realize that games, and the language embedded in game play, can be a powerful tool to help our youngsters grow to love math. This project came out of a desire to share this with other teachers, to help new teachers see the depth in game play, and that games can powerfully teach math concepts and help students develop positive attitudes toward learning math.

The experience of crafting these sessions along with having the opportunity to assist in a graduate math for teachers class built on constructivist principles has opened my eyes to the joy of working with other Bank Street graduate students. Only at Bank Street would a class have a thoughtful discussion about the merits of using dice in games. Only at Bank Street would student become teacher, teacher become student. Experiencing this math course through four different semesters, has made me realize that it's not only our youngest learners that deserve the opportunity to construct their own thinking around mathematics. Our new teachers, often entering the math course with negative and painful feelings about their own math learning, deserve to re-engage with mathematics in a positive and experiential way.

I feel very grateful to have joined Hal in guiding graduate students through this experiential journey and, in many cases, learn to love math, and love teaching math. I'm also grateful that I had the opportunity to contribute, even if in some small way, to these experiences.

In a course that exposes beginning teachers to constructivist math practices, I felt that games were an accessible and desirable medium for capturing the pleasure of doing math. During game play, the teacher is automatically removed as the source of knowledge; the teacher *must* step back to allow students to experience the game, and therefore, the concepts embedded within the game. Though I had a wonderful model of progressive education as a young student, I still often find that removing myself from the act of telling students is very difficult to do. Games are a place for a teacher to practice putting students at the center of the learning. And my hope is that these experiences can help other beginning teachers explore their role in the classroom, and help bring more play into their math classrooms.

References

Note: Not all of these sources were directly quoted in presentations or in the rationale.

Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Bell, N. (2003). *Visualizing and verbalizing*. San Luis Obispo, CA: Gander Pub.

Burns, M. (2007). *About teaching mathematics: A k-8 resource* (3rd ed.). Sausalito, CA: Math Solutions Publications

Clements, D. H., & Battista, M. T. (2002). Constructivist learning and teaching. In National Council of Teachers of Mathematics, *Putting research into practice in the elementary grades: Readings from journals of the NCTM* (pp. 6-8). Reston, VA: NCTM

Dewey, J. (1938). *Experience and Education*. United States: Kappa Delta Pi.

Hiebert, J. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Labinowicz, Ed (1980) *The Piaget Primer*

Losq, C. S. (2005). Number Concepts and Special Needs Students: The Power of Ten Frame Tiles. Teaching Children Mathematics, 310-315.

Melnick, Hal PhD, (2000, 1987) *The Concept Teaching Game: A Rationale* In Thought and Practice; the journal of the Graduate School of Education .

Volume 1, Number 1, Spr 1987.

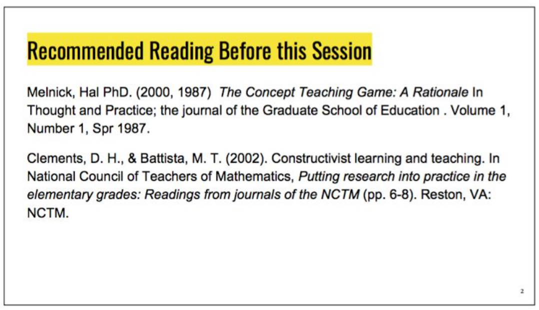
Murray, M., & Jorgensen, J. (2007). *The differentiated math classroom: A guide for teachers, K-8*. Portsmouth, NH: Heinemann.

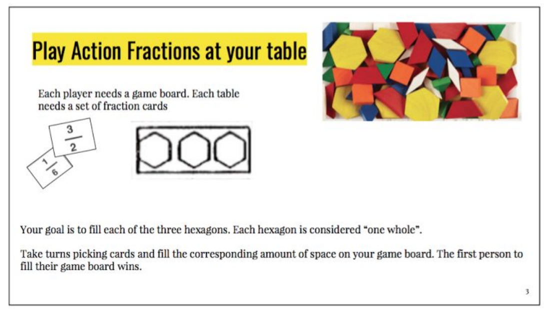
National Council of Teachers of Mathematics. *Learning and Mathematics Games, NCTM Monograph #1, 1985*

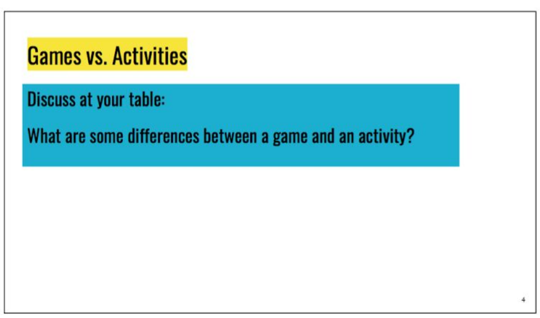
Parrish, S. (2010). *Number talks: helping children build mental math and computation strategies, grades K-5*. Sausalito, CA: Math Solutions.

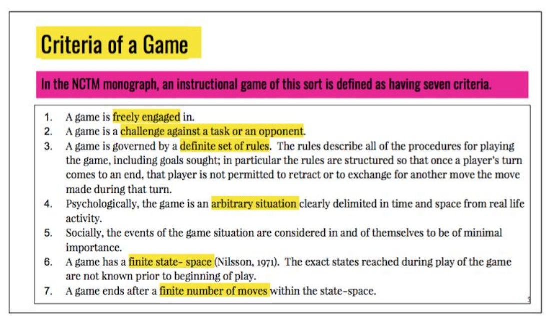
Appendix

First Experience









Discussing the Criteria

Discuss:
1. How does this list compare to what you previously came up with?
2. Look through these criteria with regards to Action Fraction.
3. Are there any criteria you particularly agree or disagree with when thinking about games in the classroom?

1. A game is freely engaged in.
2. A game is a challenge against a task or an opponent.
3. A game is governed by a definite set of rules. The rules describe all of the procedures for playing the game, including goals sought; in particular the rules are structured so that once a player's turn comes to an end, that player is not permitted to retract or to exchange for another move the move made during that turn.
4. Psychologically, the game is an arbitrary situation clearly delimited in time and space from real life activity.
5. Socially, the events of the game situation are considered in and of themselves to be of minimal importance.
6. A game has a finite state- space (Nilsson, 1971). The exact states reached during play of the game are not known prior to beginning of play.
7. A game ends after a finite number of moves within the state-space.

6

"Hopeful Moments"

A hopeful moment is when the game player expresses a desired outcome within the game. These hopeful moments, ideally, relate to the concept the game is teaching.

I hope I get a...

I hope you...

If I get ____ then ____ will happen.

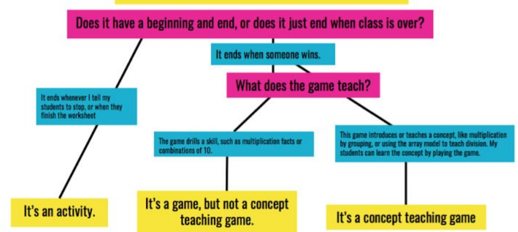
7

It may be a game, but is it a CONCEPT teaching game?

With your tablemates, compile a list of all of the things one can learn from playing Action Fraction. Who would benefit from playing this game, and why?

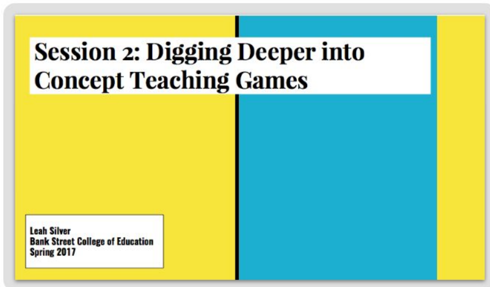
8

Is it a Concept Teaching Game?

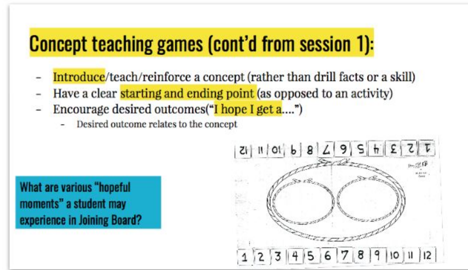


9

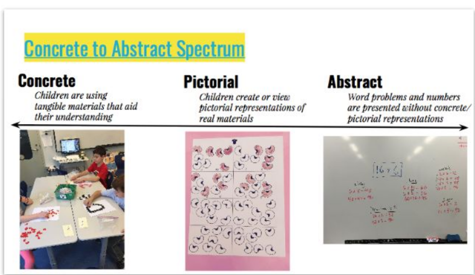
Second Experience:



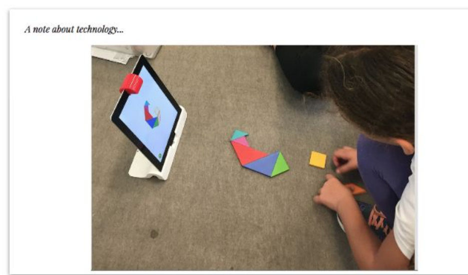
1



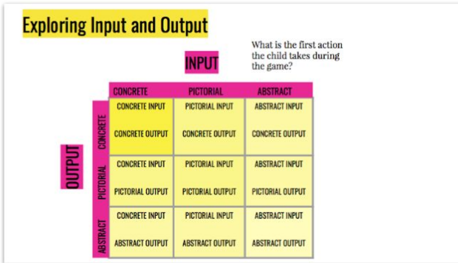
2



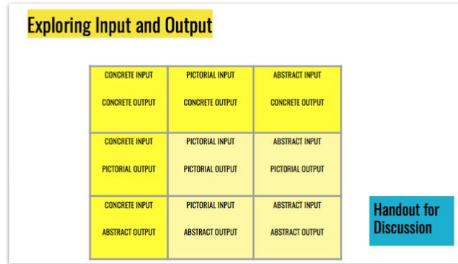
3



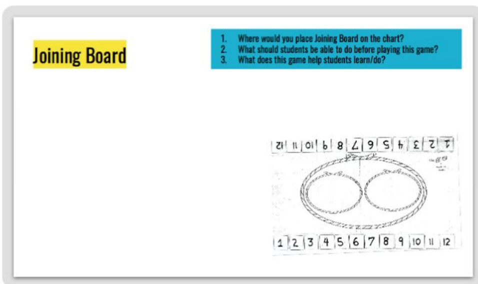
4


Exploring Input and Output
What is the first action the child takes during the game?
INPUT
OUTPUT
CONCRETE
PICTORIAL
ABSTRACT
CONCRETE INPUT
CONCRETE OUTPUT
PICTORIAL INPUT
CONCRETE OUTPUT
ABSTRACT INPUT
CONCRETE OUTPUT
CONCRETE INPUT
PICTORIAL OUTPUT
PICTORIAL INPUT
PICTORIAL OUTPUT
ABSTRACT INPUT
PICTORIAL OUTPUT
CONCRETE INPUT
ABSTRACT OUTPUT
PICTORIAL INPUT
ABSTRACT OUTPUT
ABSTRACT INPUT
ABSTRACT OUTPUT

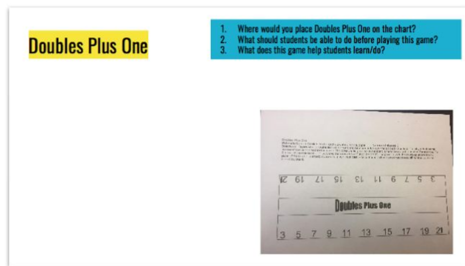
5


Exploring Input and Output
CONCRETE INPUT
CONCRETE OUTPUT
PICTORIAL INPUT
CONCRETE OUTPUT
ABSTRACT INPUT
CONCRETE OUTPUT
CONCRETE INPUT
PICTORIAL OUTPUT
PICTORIAL INPUT
PICTORIAL OUTPUT
ABSTRACT INPUT
PICTORIAL OUTPUT
CONCRETE INPUT
ABSTRACT OUTPUT
PICTORIAL INPUT
ABSTRACT OUTPUT
ABSTRACT INPUT
ABSTRACT OUTPUT
Handout for Discussion

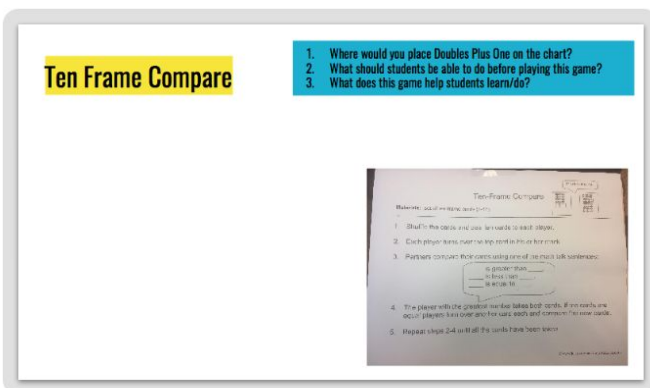
6


Joining Board
1. Where would you place Joining Board on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?
1 2 3 4 5 6 7 8 9 10 11 12

7

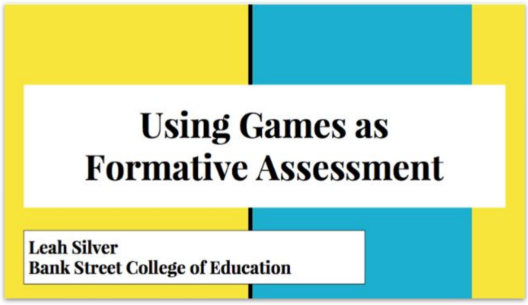

Doubles Plus One
1. Where would you place Doubles Plus One on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?
Doubles Plus One
3 5 7 9 11 13 15 17 19 21

8

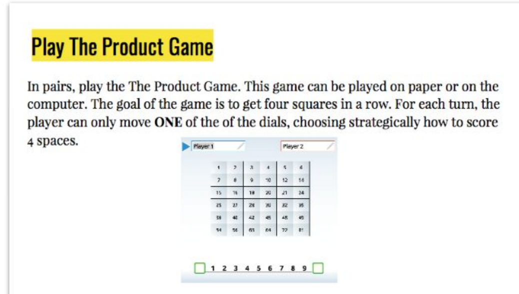

Ten Frame Compare
1. Where would you place Doubles Plus One on the chart?
2. What should students be able to do before playing this game?
3. What does this game help students learn/do?
Ten-Frame Compare

9

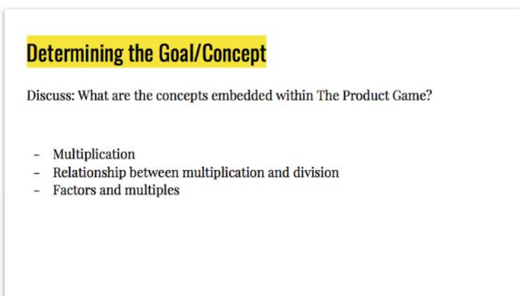
The Third Experience



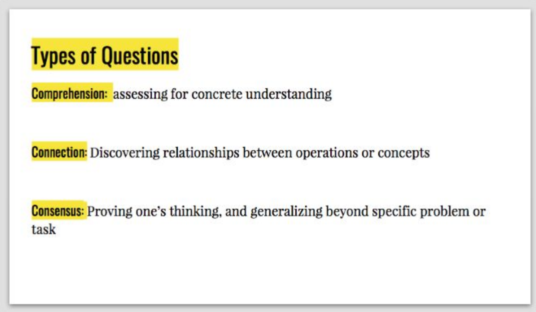
1



2



3



4

Comprehension

Short Answer	Brief response, often an answer to a closed question	What did you get?
Brief statement	More information; still not elaborate	How did you get that?
Describe	Verbalization of thinking	How did you get that?
Elaborate or clarify	Adding more information to make things clearer	What did you mean by that?
Represent	Showing thinking in one or more ways	What would that look like in a picture or manipulative?
Compare	Determining whether two strategies or ideas are the same	Is ____'s thinking the same or different that ____'s thinking?

Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Connection

Relate	Determining how strategies, ideas or representations are similar or different	How is _____'s thinking the same or different than _____'s thinking?
Discern patterns	Recognizing and describing patterns across ideas, strategies and representation	What are you noticing as you think about all these…?
Discern structure	Recognizing and describing structures across ideas, strategies and representations	What are you noticing about the way these things fit together?
Reason	Explaining why the thinking is mathematically sensible	Why do you think your thinking about your idea is true (or strategy works or representation is accurate)?
Transfer	Using thinking in a new situation or context	Can you try this in this new situation?

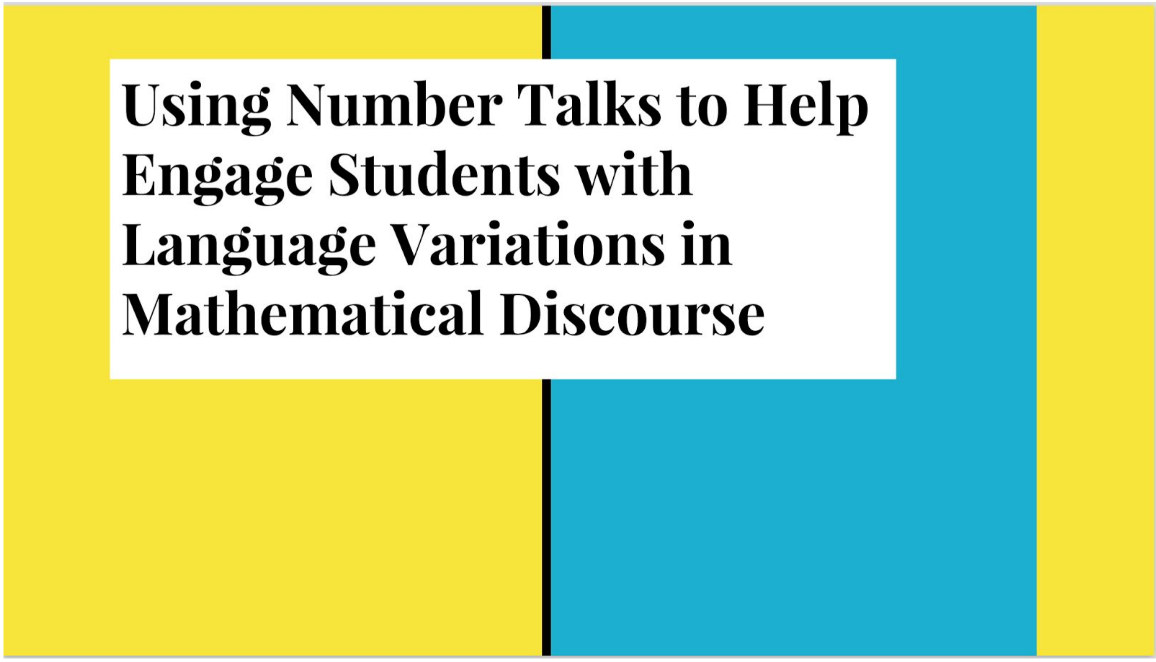
Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

Consensus

Justify	Explaining why someone else's thinking is mathematically sensible or not	Why do you think someone else's idea is true (or strategy works or representation is accurate)?
Prove	Justifying truth, workability, or accuracy within a large domain	How do you know it is true in all cases?
Refine	Stating, solving, or representing more efficiently	Can you think of a more efficient way?
Generalize	Stating the net result of a proof precisely	How would you state what you have shown to be always true?

Bahr, D., & Bahr, K. (2017). Engaging All Students in Mathematical Discourse. *Teaching Children Mathematics, 23*(6).

The Fourth Experience



COMMON LANGUAGE

SOME MATH EDUCATION TERMS EXPLAINED...

COMPUTATIONAL FLUENCY

- Three key components: accuracy, efficiency, flexibility
- Understanding the meaning of each operation and how it relates to the others
  - *Ex: Addition is the inverse of subtraction*
- Knowing number facts and relationships
  - *Ex: 2 x 5 and how it relates to 2 x 50*
- A solid understanding of our place value system as a base 10 system
  - *Ex: A '5' means something different in the tens place than it does in the 100s place (50 vs. 500)*

This definition of computational fluency was adapted from the description from the Invesitgations in Number, Data and Space curriculum, retrieved from http://investigations.terc.edu/library/bookpapers/comp_fluency.cfm.

NUMBER TALKS

- Developed by Ruth Parker and Kathy Richardson
- Posing an abstract math problem such as 18 x 5 and asking students to solve the problem mentally.
- The teacher then collects the different methods and looks at why/how they work.
- For example a teacher may pose 18 x 5 and and that students solve the problem in these different ways.

20 x 5 = 100 2 x 5 = 10 100 - 10 = 90	10 x 5 = 50 8 x 5 = 40 50 + 40 = 90	18 x 5 = 9 x 10 9 x 10 = 90	18 x 2 = 36 2 x 36 = 72 18 + 72 = 90	9 x 5 = 45 45 x 2 = 90

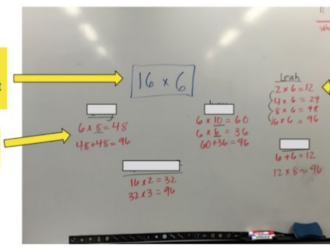
This description of *Number Talks* was adapted from Jo Boaler's article, *Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts*

A SAMPLE NUMBER TALK

This was a *Number Talk* I did with my students in the 2014-15 school year.

1. Teacher poses a problem and students think about it mentally

2. Students share various strategies



3. Sometimes the teacher shares a strategy, too!

Discuss: What language skills must a student have in order to participate in a Number Talk?

LET'S WATCH A NUMBER TALK

Click here to watch a 2nd grade Number Talk

As you watch, note the following key elements of a *Number Talk:*

- The structure (who talks when, how students share strategies, etc...)
- Teacher response to students
- How Number Talks promote computational fluency (efficiency, accuracy, flexibility

Discuss: In what ways does this structure support a child with language variations? What are the roadblocks for a student with language variations?

WHAT ARE THE POTENTIAL ROADBLOCKS FOR A STUDENT WITH LANGUAGE VARIATIONS?

OVERVIEW OF LANGUAGE PROCESSING

Two types of language; two main categories of language variations

EXPRESSIVE

What you say and how you say it

- Articulation and fluency (speech)
- Semantic use (word order, grammar structure)
- Discourse production (longer than a sentence-- narratives, conversation, etc...)

Signs of Expressive Language Variations

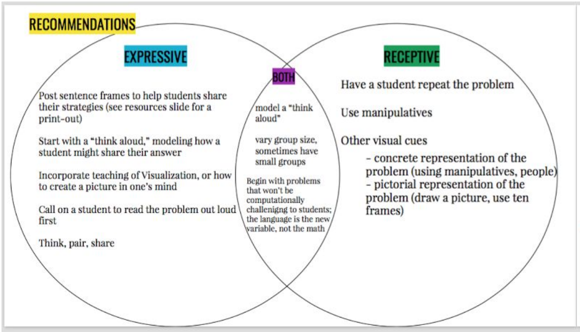
RECEPTIVE

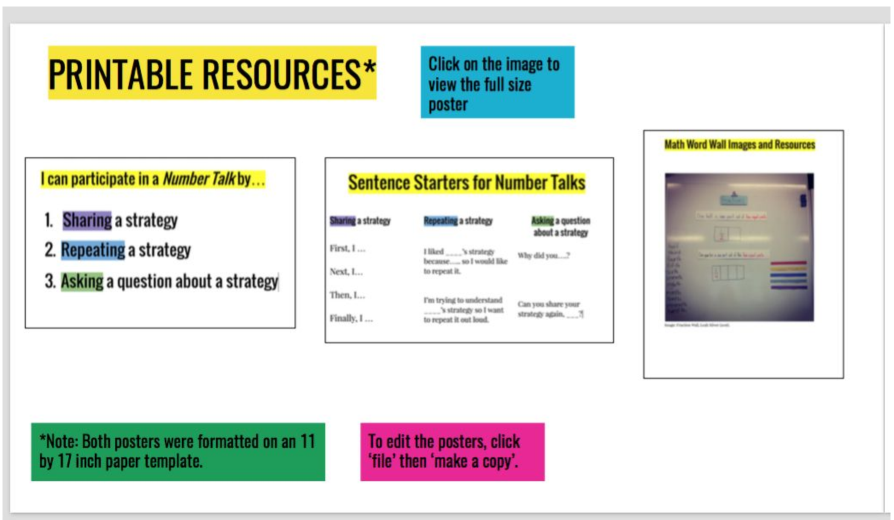
How you understand and process what you hear; All of the potential barriers a student may encounter in order to process information

- Semantic understanding
- Discourse production

Signs of Receptive Language Variations

RECOMMENDATIONS TO SUPPORT STUDENTS WITH LANGUAGE VARIATIONS





Links to Presentations:

First Experience:

https://docs.google.com/presentation/d/1Ztg1eplvul5LchYSRGiQF_fcqD_T9XMtmnIK46dA7NU/edit?usp=sharing

Second Experience:

https://docs.google.com/presentation/d/1huiX_GeMNVwtHMAaC6XW6b7scqxUKrzUPnBanOAs0KQ/edit?usp=sharing

Third Experience:

https://docs.google.com/presentation/d/1xEjvzuarj6bW2RlHHgFzPTsSRU1TBEqNMeRykdZMSdM/edit?usp=sharing

Fourth Experience:

https://docs.google.com/presentation/d/1eUsnaMiyJ5_tyCQW0dBsnA2MIGVunFl1paCUBw3WCps/edit?usp=sharing

Other Handouts

Discussion Sheet for Concrete-Abstract Spectrum

Link:https://docs.google.com/document/d/1E3HNsmtexJQuVU6olJ3TzRo4nNUCUKaacuuSjGBDN9w/edit?usp=sharing

Second Experience:
Exploring Input and Output in Concept Teaching Games

OUTPUT \ INPUT	CONCRETE	PICTORIAL	ABSTRACT
CONCRETE	CONCRETE INPUT CONCRETE OUTPUT	PICTORIAL INPUT CONCRETE OUTPUT	ABSTRACT INPUT CONCRETE OUTPUT
PICTORIAL	CONCRETE INPUT PICTORIAL OUTPUT	PICTORIAL INPUT PICTORIAL OUTPUT	ABSTRACT INPUT PICTORIAL OUTPUT
ABSTRACT	CONCRETE INPUT ABSTRACT OUTPUT	PICTORIAL INPUT ABSTRACT OUTPUT	ABSTRACT INPUT ABSTRACT OUTPUT

1. Where would you place the game on the input/output chart?

2. What experiences should a student have and what concepts should students know before playing this game?

3. What does this game help students learn/do?

Resources for Students with Language Variations (and all students):

“Ways to Participate During a Number Talk”

Link:

https://docs.google.com/document/d/1vzrEoh8JeiFt4eSMs0wgYV1ZAG6-eV-DetBN0m4U4fs/edit?usp=sharing

I can participate in a *Number Talk* by…

1. Sharing a strategy
2. Repeating a strategy
3. Asking a question about a strategy

Sentence Starters for Number Talks

Sentence Starters for Number Talks

Sharing a strategy

First, I …

Next, I…

Then, I…

Finally, I …

Repeating a strategy

I liked ____'s strategy because….. so I would like to repeat it.

I'm trying to understand ____'s strategy so I want to repeat it out loud.

Asking a question about a strategy

Why did you….?

Can you share your strategy again, ___?